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CHAPTER FOUR

FLOW-NET CONSTRUCTION

4.1 INTRODUCTION

Flow nets are a very practical and useful tool for solving many kinds of seepage and drainage problems, and those who possess skill in their construction have at their disposal an extremely powerful technique for designing safe and economical structures exposed to the damaging effects of water. This chapter contains helpful suggestions and step-by-step examples to enable the reader to draw suitable flow nets for a wide variety of conditions he may encounter. Also Section 3.3 contains detailed information about the properties and characteristics of flow nets and their many practical uses.

The thoughtful study of well-constructed flow nets can be of much benefit to the beginner because it can help him develop a general appreciation for the nature of seepage through porous media. In this text flow nets are presented for a wide variety of cross sections (nearly 100 examples are given). Also, a recent publication contains many examples of well constructed flow nets (U.S. Department of the Army, Corps of Engineers, 1986). If a flow net can be found for a cross section approximately similar to one being studied, much time can be saved because the general pattern is already known, and, as additional flow nets are constructed for approximately similar conditions, less and less time is required.

Study of the examples and the suggestions that are given in this book, together with a reasonable amount of practice, should enable engineers to learn to construct flow nets that are sufficiently accurate for many practical purposes. If the important checks described in this chapter are correctly made, flow nets will be fundamentally accurate; however, the beginner will be less

likely to develop bad habits if his work is checked by someone who can distinguish errors.

The analysis of seepage by flow nets contributes to the proper design and construction of many kinds of engineering structure. It is generally recognized that the development of important works involving seepage should be executed in the following important steps:

1. Thorough field investigations establishing soil and geologic conditions at project sites.
2. Thorough experienced evaluation of field conditions.
3. Adequate studies and analyses.
4. Economical and sound designs.
5. Adequate specifications and construction.
6. Adequate instrumentation and observation of finished works.

Flow-net studies fit primarily into step 3; however, they are also useful in the study of seepage in completed projects (step 6).

All the flow net examples in this chapter are related to steady-state seepage. Use of flow nets to study moving saturation lines is described in Sec. 3.6. Several of the examples in this chapter are for cross sections simplified to facilitate the presentation of flow net construction techniques, and do not represent recommended designs (e.g., Figs. 4.4, 4.5, 4.6, 4.7, and 4.8). All dams should be designed with positive internal drainage systems as shown in Chapter 6 (Figs. 6.7, 6.12, 6.14, 6.21, and 6.23).

4.2 GENERAL SUGGESTIONS

This section contains a number of practical time-saving suggestions about the construction of flow nets. Much time and effort can be saved in obtaining accurate flow nets if the following procedures are observed:

1. Draw the cross section to be studied on good tracing paper; turn the sheet over and construct the flow net on the reverse side. After the flow net has been completed by freehand sketching trace it on the front side or on fresh paper. Use French curves if a smooth finished product is desired for illustrative purposes. *Freehand lines do not detract from the accuracy or usefulness of a flow net if the basic rules have been properly followed.*
2. Be practical in the number of lines drawn. Do not clutter up the drawing with too many lines, but do not use so few that essential features are lost. Remember that parts of a flow net can be subdivided to any degree that is required to emphasize detail (Fig. 3.5 and Fig. 4.2d).

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3. Be practical in selecting a scale for the drawing. A scale that is too large wastes time and erasers. An 8½-by-11-in. sheet is a good size for many flow nets, although a sheet several feet wide may sometimes be needed for the analysis of a levee or dam with a wide berm or a wide upstream blanket and for groundwater studies of wide cross sections.
4. Before starting to sketch a flow net look for important boundary conditions and for prefixed flow net lines (Figs. 4.2a and 4.3a). In every flow net some flow lines and some equipotentials are established by the boundary conditions before the flow net is started. *Noting the prefixed lines helps one to get the general shape somewhat correct in the beginning.*
5. In drawing flow nets for composite sections (those having more than one permeability) look for dominating parts of a cross section. Highly pervious or highly impervious parts sometimes have a major influence on flow patterns (Figs. 4.7 and 4.8).
6. Do not overlook the overall shape while working on details and never refine a small portion of a flow net before other parts have been fairly well developed. Corrections made in one part have some influence on every other part.
7. Study the basic rules that must be carried out and follow them (Sec. 3.3).

When the basic rules are *not* followed serious errors can be made. Figure 4.1 shows two of the common errors made by beginners. Figure 4.1a shows extraneous equipotentials, both upstream and downstream from the structure, and Figure 4.1b shows flow lines disappearing into the boundary below the permeable formation. By studying a few correctly constructed flow nets such errors should be eliminated (i.e., Figs. 4.2d and 4.3e).

4.3 TYPES OF FLOW NET

Flow nets can be of several types, depending on the configuration and the number of zones of soil or rock through which seepage is taking place. A primary subdivision can be made that depends on the following conditions:

1. *Flow is confined* within known saturation boundaries and the phreatic line is therefore known.
2. *Flow is unconfined* and the upper level of saturation (the phreatic line) is *not* known.

A second subdivision can be made that depends on whether (a) the cross section can be drawn as one zone or unit of a single permeability or (b) the cross section contains two or more zones or units of different permeabilities. The latter is described in this book as a *composite section*.

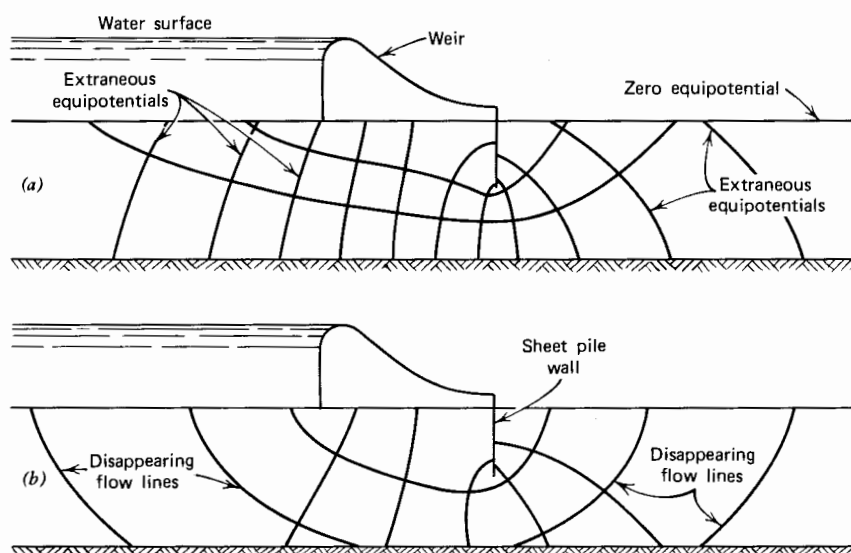


FIG. 4.1 Some common errors in flow nets by beginners. (a) Extraneous equipotentials. (b) Disappearing flow lines.

These criteria give four possible combinations of flow conditions:

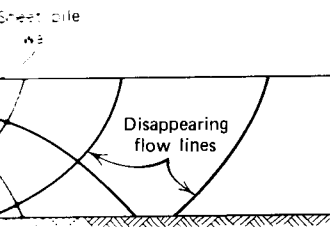
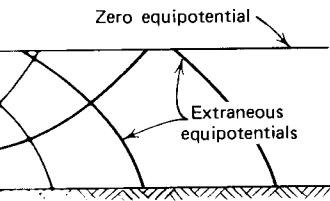
- 1a. *Confined flow in single permeability sections.*
- 1b. *Confined flow in composite sections (those having two or more permeabilities).*
- 2a. *Unconfined flow in single permeability sections.*
- 2b. *Unconfined flow in composite sections.*

Any of these types can be constructed for *isotropic* soil (horizontal and vertical permeabilities equal) or for *anisotropic* soil (horizontal and vertical permeabilities *not* equal). The rule for correcting for anisotropic conditions in which a single transformation factor is applied to an entire cross section is given in Chapter 3 (Sec. 3.3).

Step-by-step procedures for constructing flow nets are discussed in the remaining part of this chapter, starting with the simplest type and progressing to the more complex.

4.4 CONFINED FLOW SYSTEMS (Phreatic Line Known)

Example 1 *Seepage Under Sheet Pile Cutoff Wall (single permeability section).* Figure 4.2 shows a vertical sheet pile wall in a sandy foundation. This example represents type 1a flow because all seepage boundaries are defined in



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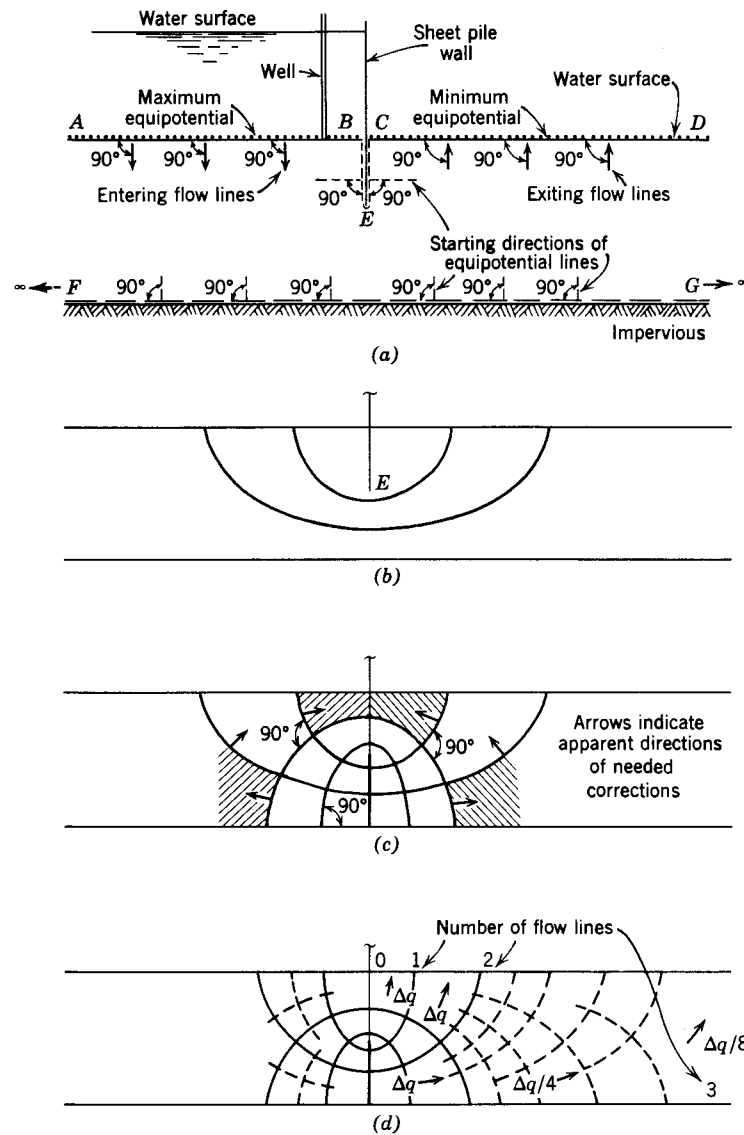


FIG. 4.2 Example of the construction of a flow net for confined flow in single permeability section (Example 1). (a) Identify prefixed conditions, noting starting directions of lines. (b) Draw trial family of flow lines (or equipotentials) consistent with prefixed conditions. (c) Keeping the lines drawn in (b), sketch first trial flow net. Make all lines intersect other set of lines at 90°. (d) Erase and redraw lines until all figures are square. Subdivided as desired for detail and accuracy.

advance of the flow net construction and seepage is restricted to the uniform sand layer.

By inspection of the cross section (Fig. 4.2a) the known flow net lines are the following:

- Line *AB*: The maximum equipotential line.
- Line *CD*: The minimum equipotential line.
- Line *BEC*: The shortest flow line.
- Line *FG*: The longest flow line.

With the knowledge that equipotential lines and flow lines must intersect one another at right angles, the directions of lines in the flow net are established at certain points. Thus all flow lines must enter the foundation at right angles to line *AB* and exit perpendicular to the line *CD*; also, equipotential lines must intersect lines *BE*, *EC*, and *FG* at right angles. The directions of these intersecting lines are shown at several random points in Figure 4.2a. Though the positions of the lines shown do not necessarily coincide with any that will be developed in the flow net, their directions give useful clues regarding the shape of the completed flow net. With the known restrictions in mind and knowing that all the lines will begin to curve soon after leaving fixed boundaries, the sketching can be started.

Flow nets can be developed in a number of different ways, including the following:

1. Immediately sketching both flow lines and equipotential lines, working from one part of a flow net into another.
2. Sketching a plausible family of flow lines before starting to sketch equipotentials.
3. Sketching a plausible family of equipotential lines before starting to sketch flow lines.
4. Combinations of the above or any other suitable procedures.

In Figure 4.2b the flow net is started by drawing two intermediate flow lines, making the entrance and exit angles 90° and keeping in the mind that seepage tends to concentrate at a focal point such as the bottom of the wall at point *E*. The next step (Fig. 4.2c) is to draw a family of equipotentials, *making all intersections with flow lines right angles and trying to make some of the figures squares*. This procedure, applied to this example, gives the first trial flow net (Fig. 4.2c). On cursory examination this flow net might appear to be fairly satisfactory, but a detailed examination shows that a number of the figures are not squares. If all intersections are 90° angles, the figures will be rectangular, as desired; however, if diamonds or other off-shaped figures are formed, lines should be deliberately shifted until all intersections are 90° .

angles and all figures are basically rectangular in shape. The figures that are not squares and the apparent directions in which lines need to be shifted to convert the figures into squares should now be noted. The word "apparent" is used because we cannot always be sure at this stage just which of the initial adjustments will prove to be correct, since the shifting of any line alters the amount of the subsequent corrections required of intersecting lines. In Figure 4.2c some of the figures that definitely are not squares are crosshatched. The apparent directions of needed corrections of this flow net are indicated by the arrows.

By repeatedly erasing and redrawing portions of flow lines and equipotential lines and studying the overall pattern and individual parts a finished flow net can be developed (Fig. 4.2d). Portions of this flow net have been subdivided one or more times to give greater detail. (See also Fig. 3.5.)

Sometimes, when drawing flow nets, it is desirable to erase all lines, leaving only a trace of the most recent for redrawing and improving the entire flow net.

If a flow net has a whole number of flow channels (2.0, 3.0, 4.0, etc.), the corresponding number of equipotential spaces depends on the shape of the cross section and will not necessarily be a whole number. The finished flow net in Figure 4.2d has a whole number of flow channels and equipotential spaces; but if the shape had been slightly different, the number of equipotential spaces might have been 5.6, 6.5, or some other fractional number.

Likewise, if a whole number of equipotential spaces was selected for a flow net, the number of flow channels could be a fractional number. In this example (Fig. 4.2) a slightly different shape could have produced 2.7, 3.5, or some other fractional number of flow channels. The flow net in Figure 4.4c with 1.2 flow channels is an example.

There is no objection to the construction of flow nets with fractional numbers of flow channels and equipotential spaces, but usually one or the other should be made a whole number.

Example 2 Flow Beneath Concrete Weir (single permeability section). Figure 4.3 is another example of confined flow in a section of one permeability (type 1a). Here, as in Example 1, all seepage boundaries are known before the flow net is started. The known flow net lines are the following (Fig. 4.3a).

Line <i>AB</i> :	The maximum equipotential line.
Line <i>CD</i> :	The minimum equipotential line.
Line <i>BEFGHC</i> :	The upmost flow line.
Line <i>IJ</i> :	The longest flow line.

Known directions of lines in the flow net that intersect these prefixed lines at right angles are shown at arbitrary points in Figure 4.3b.

In Figure 4.3c the flow net is started by drawing two intermediate flow lines,

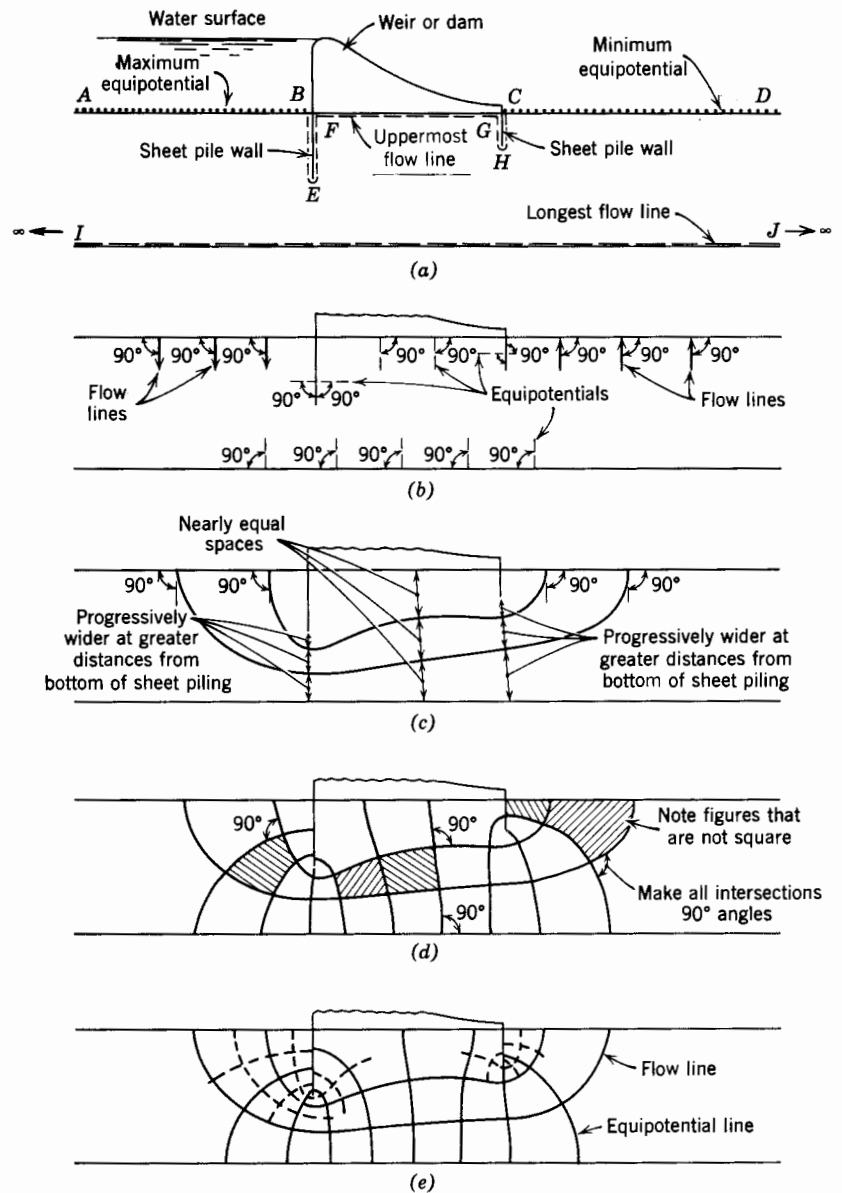
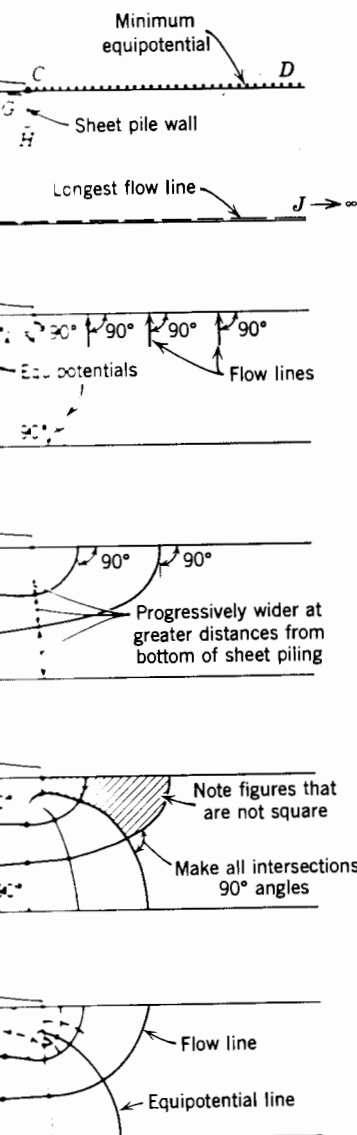


FIG. 4.3 Another example of a method of constructing flow nets with all boundaries known (Example 2). (a) Identify prefixed flow lines and equipotential lines. (b) Look for prefixed starting directions of lines. (c) Draw trial family of flow lines (or equipotentials) consistent with prefixed conditions. (d) Keeping the lines drawn in (c), sketch first trial flow net. Make all lines intersect other set of lines at 90° angles. (e) Erase and redraw lines until all figures are squares. Subdivide as desired for detail and accuracy.



Constructing flow nets with all boundaries as equipotential lines and equipotential lines. (b) Look for a family of flow lines (or equipotential lines) drawn in (c), sketch first: equipotential lines at 90° angles. (e) Erase and redraw as desired for detail and accuracy.

noting that flow radiates around the bottoms of the cutoff walls and that the lines tend to be somewhat equally spaced under the centers of wide structures.

In Figure 4.3d a trial family of equipotentials has been added to obtain a trial flow net, and several nonsquare figures have been crosshatched. The flow net obtained by progressive correction of errors is given in Figure 4.3e.

4.5 UNCONFINED FLOW SYSTEMS (Phreatic Line Unknown)

In the examples given in Sec. 4.4 the line of saturation, or *phreatic* line, is known in advance, which makes the procedure for obtaining the flow nets simpler than it is for cross sections in which the upper saturation line is not known. Cross sections with an unknown phreatic line are considered the most challenging because the phreatic line must be located simultaneously with the drawing of the flow net. The procedures described in this section permit flow nets of this kind to be constructed with minimum time and effort.

Example 3 Seepage Through Homogeneous Earth Dam. The cross section and known conditions for a type 2a flow net are given in Figure 4.4a. Line AB , the face of the dam, is the maximum equipotential line; line AC , the base of the dam, is a flow line. The exact position of the phreatic line is unknown, but it can reasonably be expected to lie somewhere within the shaded zone BDE . The general condition at the free surface is known (Fig. 3.6) and it is known that the net must be composed of squares.

Before starting to construct a flow net with an unknown phreatic line, the total head h should be divided into a convenient number of equal parts Δh , and light guidelines should be drawn across the region in which the phreatic line is expected to lie. In Figure 4.4a four intermediate guidelines (for convenience, called *head* lines in this section) are drawn at a vertical spacing $\Delta h = h/5$.

The conditions that establish the position of the phreatic line in Fig. 4.4 are the following:

1. Equal amounts of head must be consumed between adjacent pairs of equipotential lines.
2. Equipotential lines must intersect the phreatic line at the correct elevation.
3. To satisfy requirements 1 and 2, each *equipotential* line must intersect the phreatic line at the appropriate *head* line. this key requirement must be satisfied by all flow nets having an unknown phreatic line.

After drawing the horizontal guidelines or *head* lines across the region in which the phreatic line is expected to lie a *trial saturation line* (line ab , Fig. 4.4b) should be drawn; make a reasonable guess about its probable location

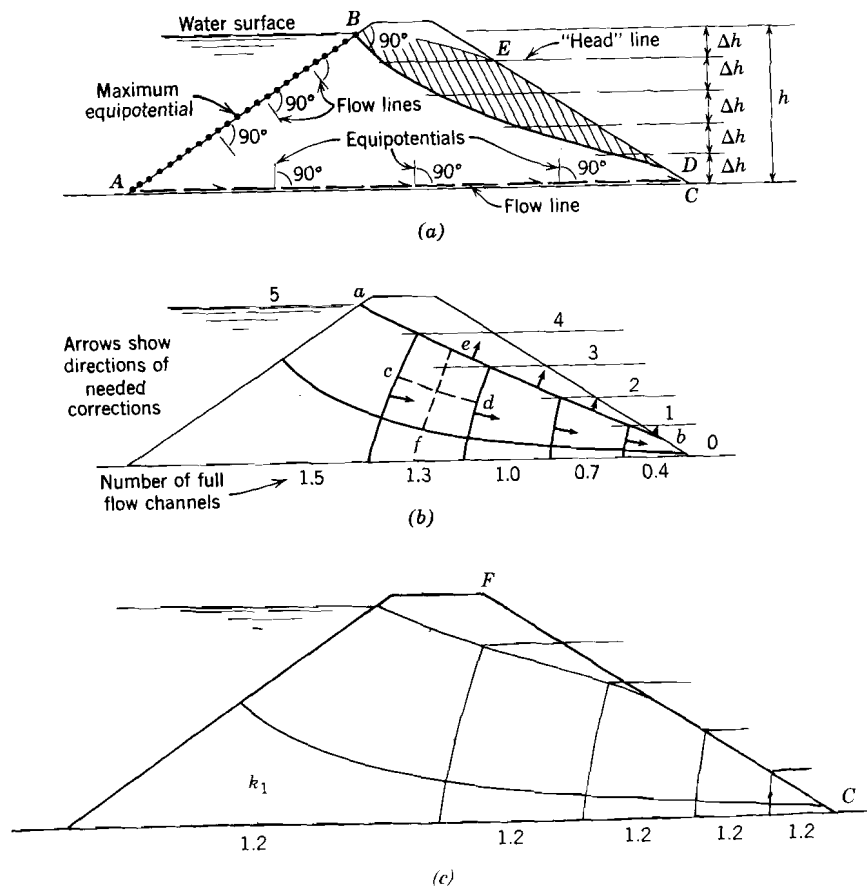


FIG. 4.4 Flow net for homogeneous dam on impervious foundation (Example 3). (a) Known conditions. (b) Trial saturation line and flow net. (c) Final flow net.

and shape. Next a family of equipotential lines should be drawn, making all intersections with flow lines right angles. Then draw one or more intermediate flow lines to establish a trial flow net as shown in Figure 4.4b. If all intersections are right angles, the first trial net will be composed of rectangular figures. Some of the figures may happen to be squares, but most will probably be elongated rectangles, as in this example.

Usually it is possible by inspection alone to note the kinds of correction that must be made and to develop accurate flow nets with an indeterminate phreatic line (e.g., Fig. 4.4c). At this point it is well to review a systematic checking method that takes the guesswork out of flow nets for unconfined seepage. This check is based on the rule: *In any flow net the number of flow channels must remain the same throughout the net.* This rule simply states that all the water entering a cross section must flow through the section and emerge

at the low potential side. It applies to sources feeding water into them or

If a flow net has been correctly constructed, the rule given here is observed. In Figure 4.3e, which has two equipotential lines. To satisfy the rule, the flow lines must encompass the same number of equipotentials. This is usually a true square, for it is a fundamental requirement if its area is to be equal to the area of the flow net. In Figure 4.3e each pair of equipotential lines and the basic rule is met. A flow net that is not a square, as in Fig. 4.4b; another that does not satisfy the rule is shown in Fig. 4.4c. Because the last two flow nets are not squares, it is possible to detect inaccuracies and correct them in two ways.

1. By inspection note the figure of the flow net. Shift lines in directions that appear to be necessary. Figure 4.4b shows initial adjustments. If this procedure is followed, the flow net will become progressively more a square and has the same number of flow channels. If you are not confident enough to use the second method.

2. Use an engineer's scale to measure the width of a trial flow net and calculate the number of flow channels. A figure that has a width of 1.0 flow channel; one that has a width of 1.3 flow channels, and so on. Thus in Figure 4.4b the ratio of ef to cd is 1.0; hence one full flow channel and a fractional space below this square. Therefore, the number of flow channels is 1.0 + 0.3, or 1.3. This number is written down. A similar procedure of measuring the width of the flow channels is carried out for the balance of the flow net. In Figure 4.4b the numbers written down are 1.2, 1.2, 1.2, 1.2, 1.2. If the numbers written down are not in balance and must be redrawn. The trial flow net (Fig 4.4b) these numbers are adjusted. The adjustments indicated by the arrows are to raise the phreatic line above the trial flow net, to raise the phreatic line above a given point and to lower the phreatic line above that point; if the figure is too low above the point.

To correct a trial flow net that

at the low potential side. It applies to all sections that have no secondary sources feeding water into them or drains removing water from them.

If a flow net has been correctly constructed and is *composed entirely of squares*, the rule given here is observed automatically. An example is the flow net in Figure 4.3e, which has two intermediate flow lines and eight intermediate equipotential lines. To satisfy this rule each adjacent pair of equipotential lines must encompass the same number of "square" figures, a square being a figure that meets the requirement described in Sec. 3.3. A "square" is not usually a true square, for it is enclosed in curved lines, but *it satisfies the fundamental requirement if its average width equals its average length*. In Figure 4.3e each pair of equipotential lines encompasses three squares; hence the basic rule is met. A flow net that does *not* fulfill this rule is the trial flow net in Fig. 4.4b; another that *does* fulfill it is the finished flow net in Figure 4.4c. Because the last two flow nets are somewhat similar in appearance, how is it possible to detect inaccuracies and make the necessary corrections? It can be done in two ways.

1. By inspection note the figures in a trial flow net that are *not* squares and shift lines in directions that appear reasonable; for example, the arrows in Figure 4.4b show initial adjustments that are judged from visual inspection to be necessary. If this procedure is correctly applied, subsequent trial flow nets will become progressively more accurate until *the flow net is composed of squares and has the same number of flow channels throughout* (Fig. 4.4c). If you are not confident enough to improve flow nets by inspection alone, then use the second method.

2. Use an engineer's scale to measure the widths and lengths of figures in a trial flow net and calculate the number of flow channels between pairs of equipotentials. A figure that has a width-to-length ratio of 1.0 contains 1.0 flow channel; one that has a width-to-length ratio of 0.5 contains 0.5 flow channel, and so on. Thus in Figure 4.4b lines *cd* and *ef* are equal and the ratio of *ef* to *cd* is 1.0; hence one full flow channel is contained in this figure. the fractional space below this square has a width-to-length ratio of about 0.3; therefore, the number of flow channels between equipotentials 3 and 4 is $1.0 + 0.3$, or 1.3. This number is written below the base of the dam as shown. A similar procedure of measuring figures and recording the total number of flow channels is carried out for the balance of the flow net, as indicated in Figure 4.4b. If the numbers written down are not all nearly equal, the flow net is out of balance and must be redrawn until these numbers are the same. In this trial flow net (Fig 4.4b) these numbers are not at all equal. The need for the adjustments indicated by the arrows is now apparent. This adjustment will raise the phreatic line above the trial position in Figure 4.4b. Generally, if the "squares" above a given point are elongated horizontally, the phreatic line is *too low* above that point; if the figures are elongated vertically, it is *too high* above the point.

To correct a trial flow net that has an indeterminate phreatic line the line

should be raised if the first guess is too low or lowered if the first guess is too high. This correction automatically forces the equipotential lines to be moved in the correct direction. After the phreatic line is raised or lowered, a new family of equipotentials should be drawn and the new flow net, examined. If the figures are not all squares and the calculated number of flow channels is not the same throughout, the procedure should be repeated until a satisfactory flow net is obtained (Fig. 4.4c). With experience accurate flow nets can be developed with the help of visual inspection alone. If any doubt exists, the lengths and widths of the figures should be measured with an engineer's scale and the number of flow channels calculated at several places in the section, as described in the preceding paragraph. If this check is properly made, accuracy is guaranteed. As a flow net is improved and all figures become true squares, the phreatic line will be forced into its correct position. In developing this kind of flow net we are solving simultaneously for the flow net and the position of the phreatic line.

When seepage emerges along a sloping surface, such as the downstream slope of a dam (Fig. 4.4c), some of the water flows down the slope and a portion of the flow net is thereby cut off. When counting the number of flow channels in the extremities of flow nets, cut-off portions should be included in the totals. Thus in Figure 4.4c the finished flow net has 1.2 flow channels throughout, even though a portion of the right end of the flow net is cut off. If line *CF* in Figure 4.4c were a boundary between zones of two different permeabilities k_1 to k_2 , the shape of the flow net would depend on the ratio of k_2 to k_1 . If the second zone is considerably more permeable than the first ($k_2/k_1 = 1000$ or more), the influence of the second zone on the flow net can usually be ignored. If the ratio k_2/k_1 is in a moderate range, several hundred or less, the method described in Sec. 4.6 for *composite sections* should be used in developing the flow net.

4.6 COMPOSITE SECTIONS WITH PHREATIC LINE UNKNOWN

In this section a method of checking the accuracy of flow nets for composite sections with unknown phreatic lines is described and examples of shortcut methods are given.

Example 4 Zoned Earth Dam. Figure 4.5 illustrates a basic procedure for drawing composite flow nets for sections with unknown phreatic lines. The transfer conditions at boundaries between soils of different permeability materials were described in Sec. 3.3. When water flows across a boundary into a soil of different permeability, the figures in the second soil must elongate or shorten to make the length-to-width ratios of the figures satisfy Eq. 3.11:

$$\frac{c}{d} = \frac{k_2}{k_1} \quad (3.11)$$

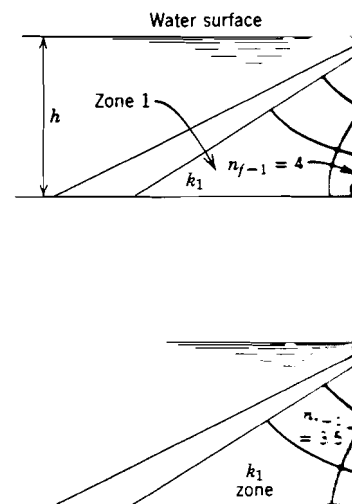


FIG. 4.5 Method for constructing a flow net for a composite section. First trial flow net (not correct).

In Eq. 3.11 c is the length and d is the width of the figures in the second soil, and k_2 is the permeability of the first soil.

Figure 4.5 shows a common type of composite section with a sloping phreatic line. It is for a dam with a permeable upstream face and an impervious downstream face. The permeability in the upstream face is k_2 and the permeability in the downstream face is k_1 . The phreatic line is a type $2b$ (Sec. 4.3) because it is a boundary between two zones. In this example the downstream face is the upstream zone.

To develop a flow net for this section, the following steps are followed:

1. Locate the *reservoir level* at the upstream face as h and dividing h into increments Δh . Draw a series of equipotential lines at intervals of Δh across the section.
2. *Guess* a trial position for the phreatic line. Draw a preliminary flow net as shown in Figure 4.5. The rectangles in zone 2 are drawn to approximate the phreatic line, using an assumed width of the figures as in Figure 4.5. When completed, the trial flow net is examined to see if it satisfies the basic shape requirements. The length-to-width ratio of the shapes in zone 2 must satisfy Eq. 3.11.

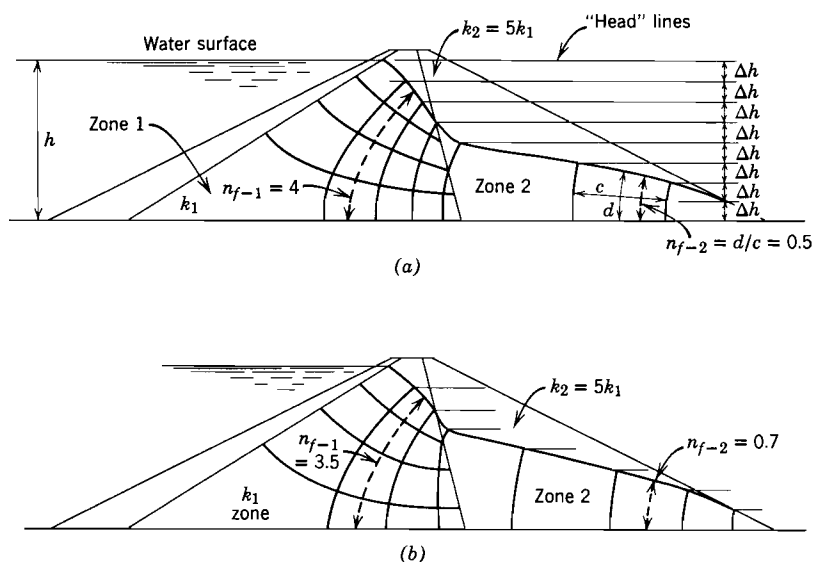


FIG. 4.5 Method for constructing flow nets for composite sections (Example 4). (a) First trial flow net (not correct). (b) Completed flow net.

In Eq. 3.11 c is the length and d , the width of the figures in the second soil; the permeability of the first soil is k_1 , that of the second k_2 .

Figure 4.5 shows a common type of composite cross section with an indeterminate phreatic line. It is for a zoned earth dam with a soil of slightly lower permeability in the upstream part than in the downstream part. This section is a type 2b (Sec. 4.3) because it is for unconfined flow in a composite section. In this example the downstream zone is assumed to be five times as permeable as the upstream zone.

To develop a flow net for this type of section these steps should be followed.

1. Locate the *reservoir level* and the *tail water level*, noting the difference in head as h and dividing h into a convenient number of equal parts or increments Δh . Draw a series of light horizontal guide lines (*head lines*) at intervals of Δh across the downstream part of the section (Fig. 4.5a).
2. *Guess* a trial position for the phreatic line in both zones and draw a preliminary flow net as shown in Figure 4.5a, making squares in zone 1 and rectangles in zone 2. Make the length-to-width ratios of all of the rectangles in zone 2 approximately equal by adjusting the shape of the saturation line, using an engineer's scale to measure the lengths and widths of the figures as in Example 3, Figure 4.4. When this step is completed, the trial flow net should be reasonably well drawn. It should satisfy the basic shape requirements of a flow net, but the length-to-width ratio of the shapes in zone 2 probably will not satisfy Eq. 3.11.

Although the flow net has been drawn for a composite section, the ratio of k_2/k_1 probably does not equal the k_2/k_1 ratio originally assumed for the section.

3. Calculate the actual ratio of k_2/k_1 for the trial flow net just constructed. To make this important check proceed as follows:
 - (a) Count the number of full flow channels between any two adjacent equipotential lines in zone 1 and call this number n_{f-1} . In the trial flow net in Figure 4.5a $n_{f-1} = 4.0$.
 - (b) Count the number of full flow channels between any two adjacent equipotential lines in zone 2 and call this number n_{f-2} . In Figure 4.5a n_{f-2} is equal to the width-to-length ratio of the figures in zone 2, d/c and equal to 0.5.
 - (c) The actual value of k_2/k_1 for the trial flow net in Figure 4.5a can now be determined from the equation

$$k_2 = k_1 \frac{n_{f-1}}{n_{f-2}}$$

or

(4.1)

$$\frac{k_2}{k_1} = \frac{n_{f-1}}{n_{f-2}}$$

- (d) If the calculated k_2/k_1 ratio is *too high*, the saturation line in zone 2 is too low and must be raised. If the calculated k_2/k_1 ratio is *too low*, the saturation line in zone 2 is too high and must be lowered. *Raise* or *lower* the general level of the saturation line in zone 2 as indicated and construct another trial flow net.
- (e) Repeat steps (a) through (d) until a flow net of the desired accuracy is obtained. (Usually a few trials will be sufficient.)

By applying Eq. 4.1 to the first trial flow net in this example (Fig. 4.5a) $k_2/k_1 = 4.0/0.5 = 8.0$. Because the ratio of k_2/k_1 for this example was assumed to be 5, the k_2/k_1 ratio of the trial flow net is *too high*; hence the general level of the saturation line in zone 2 is too low and must be raised. For the second trial flow net (Fig. 4.5b) $n_{f-1} = 3.5$ and $n_{f-2} = 0.7$. The *calculated ratio* of $k_2/k_1 = 3.5/0.7 = 5.0$, the value originally assumed.

Equation 4.1 may be derived by recalling that the quantity of seepage in zones 1 and 2 (Fig 4.5) must be equal. Using Eq. 3.18,

$$q = kh \frac{n_f}{n_d} \quad (3.18)$$

in zone 1, $q = k_1 h (n_{f-1}/n_d)$, and in zone 2, $q = k_2 (n_{f-2}/n_d) \sim k_2 (n_{f-2}/n_d)$ and $n_{f-2} = d/c$ and $k_2 = k_1 (n_{f-1}/n_{f-2})$. The permeability ratios k_2/k_1 are being examined for accuracy. The fundamental relationships must be kept in mind when flow nets for composite sections are constructed.

More than one system of lines is illustrated by Figure 4.6 which groups the lines as shown in Figure 4.5. Although the lines are not actually identical. The total head is 20 ft. As drawn in Figure 4.6a, the solid lines that form squares, the

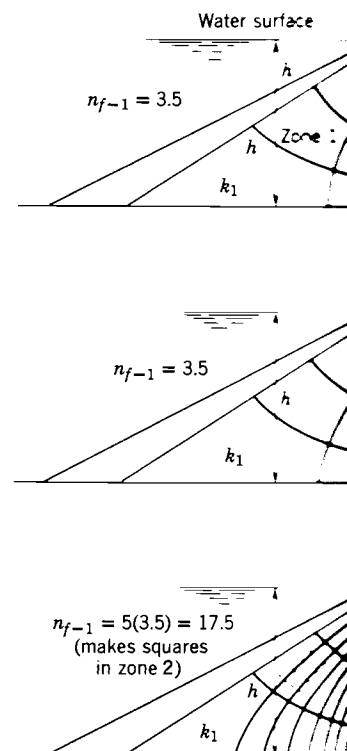


FIG. 4.6 The

in zone 1, $q = k_1 h(n_{f-1}/n_d)$, and in zone 2, $q = k_2 h(n_{f-2}/n_d)$. For a given head h , $q \sim k_1(n_{f-1}/n_d) \sim k_2(n_{f-2}/n_d)$ and $q/n_d \sim k_1 n_{f-1} \sim k_2 n_{f-2}$. Therefore $k_1 n_{f-1} = k_2 n_{f-2}$ and $k_2 = k_1(n_{f-1}/n_{f-2})$. This expression (Eq. 4.1) can be used for determining the permeability ratios k_2/k_1 , k_3/k_2 , and so on, for any composite flow net being examined for accuracy. It is an essential criterion to be used in constructing accurate flow nets for composite sections.

The fundamental relationships represented in Eqs. 3.11 and 4.1 should be kept in mind when flow nets for composite sections are being studied or constructed.

More than one system of lines can be selected for a flow net. This point is illustrated by Figure 4.6 which gives three sets of lines for the flow net developed in Figure 4.5. Although the three nets appear to be different, they are actually identical. The total head h has been divided into eight parts equal to Δh . As drawn in Figure 4.6a, the portion in zone 1 has been constructed with solid lines that form squares, the portion in zone 2 with elongated rectangles

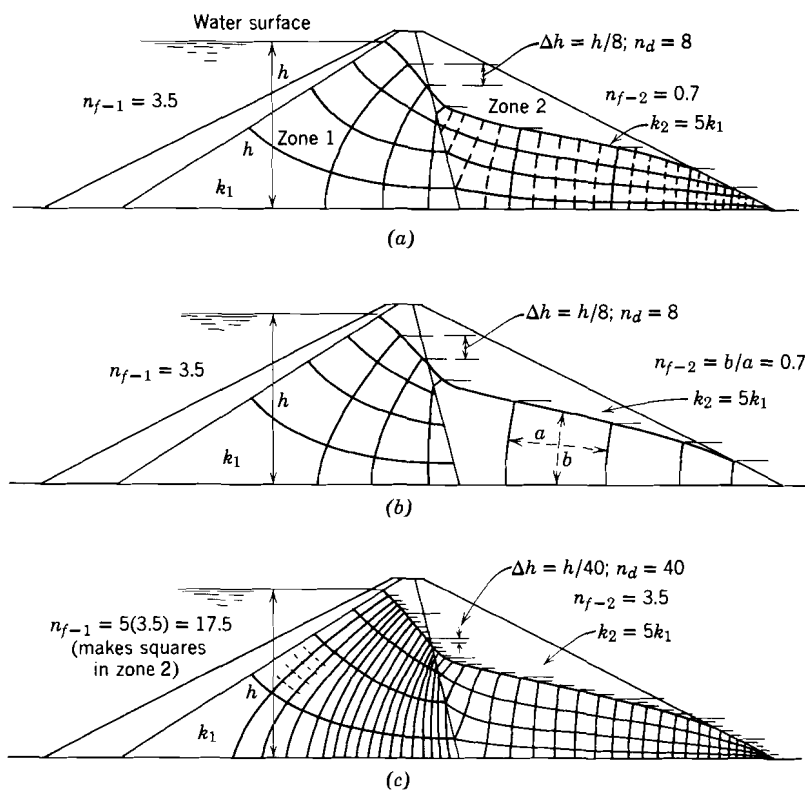


FIG. 4.6 Three forms of one flow net.

(solid lines) with length-to-width ratios of 5. The dashed lines in zone 2 divide the elongated rectangles into squares. For this flow net $n_{f-1} = 3.5$, $n_{f-2} = 0.7$, and $n_d = 8$. The seepage quantity q can be computed by using k_1 or k_2 . Using k_1 , $q = k_1 h (n_{f-1} / n_d) = k_1 h (3.5 / 8) = 0.44 k_1 h$. Using k_2 , $q = k_2 h (n_{f-2} / n_d) = k_2 h (0.7 / 8) = 0.088 k_2 h$, but $k_2 = 5k_1$, and $q = 0.088(5k_1)h = 0.44 k_1 h$.

The line system in Figure 4.6a is typical of that frequently used for this type of flow net. The dashed lines may be omitted unless they are needed in stability studies or to bring out detail.

Sometimes all intermediate lines in one or more zones may be omitted, as in zone 2, Figure 4.6b. In this flow net, as in Fig. 4.6a the head h is divided into eight parts equal to Δh and the number of equipotential drops $n_d = 8$. In zone 1 the number of flow channels $n_{f-1} = 3.5$; in zone 2 the number of flow channels $n_{f-2} = 0.7$ (as in Fig. 4.6a). Seepage quantities determined by using k_1 and k_2 are identical to those made from the flow net in Figure 4.6a.

A third system of lines for this flow net shown in Figure 4.6c was obtained by dividing the total head h into 40 parts, each one-fifth as large as Δh in Figures 4.6a and 4.6b. The flow net in Figure 4.6c has been constructed with squares in zone 2 and shortened rectangles in zone 1. Because $k_2 = 5k_1$, the figures in zone 1 must be shortened to lengths equal to one-fifth of their widths. This relationship can be verified readily by Eq. 3.11. With the flow net drawn as shown in Figure 4.6c, $n_{f-2} = 3.5$ and $n_{f-1} = 3.5(5) = 17.5$, for it is determined by the number of squares between equipotentials. The number of equipotential drops $n_d = 40$. Computing the seepage quantity in zone 1, using k_1 , $q = k_1 h (n_{f-1} / n_d) = k_1 h (17.5 / 40) = 0.44 k_1 h$, which was obtained from the flow net drawn in Fig. 4.6a. Computing the seepage quantity in zone 2, using k_2 , $q = k_2 h (n_{f-2} / n_d) = k_2 h (3.5 / 40) = 0.088 k_2 h$, which was also obtained for the flow net drawn in Figure 4.6a. Recalling that $k_2 = 5k_1$, $q = 0.088(5k_1)h = 0.44 k_1 h$, as obtained before.

By inspection of the three forms of the flow net in Figure 4.6 we see that the flow patterns and hydraulic gradients are identical in all three. This should be true because this is but one flow net.

If a flow net is being constructed for a section in which k_2/k_1 is large, it may not be practical to subdivide the elongated rectangles in the manner described above. In such cases (Sec. 3.3) the measurement of the lengths and widths of flow-net figures with an engineer's scale can establish the accuracy of basic shapes.

Example 5 Earth Dam on Low Permeability Foundation. Figure 4.7 shows an earth dam (or levee) on a foundation that is relatively low in permeability in relation to the embankment. In this problem it is assumed that the permeability of the foundation k_f is one-tenth of the permeability of the embankment k_e . When seepage occurs through two dissimilar but more or less parallel zones, as in Figure 4.7, flow through the more permeable zone dominates the combined seepage pattern. Time can often be saved by constructing a flow net for the more permeable part, the dam in this case, assuming temporarily that

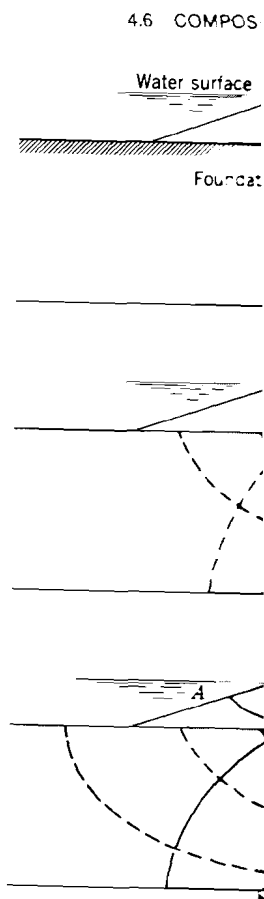


FIG. 4.7 Development of flow net for dam. (a) Construct flow net for dam. (b) Extend equipotential lines into foundation. (c) Adjust equipotentials and flow lines.

the other part is completely impermeable. In the completed flow net, a balanced flow net is developed (Fig. 4.7c). In nets of this kind, fundamental rules must be kept in mind. In the completed flow net, the foundation conducts only one-tenth of the seepage of the more permeable embankment. The equipotential lines are shown as dashed lines between the dam and the foundation. The embankment in Figure 4.7a is shown as a solid line. However, the first full flow channel is shown as a solid line. Seepage beneath this flow line is neglected, thus raising the total value

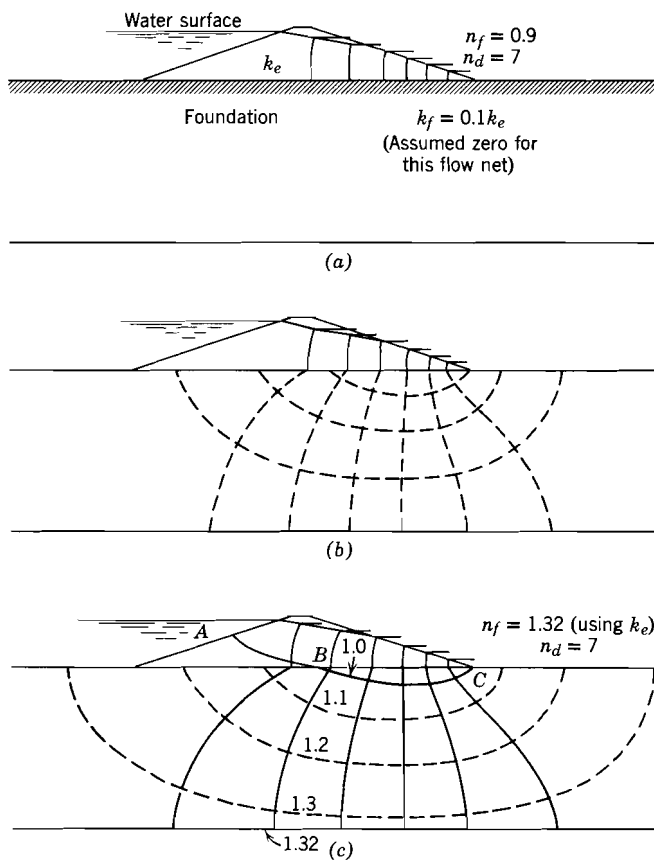


FIG. 4.7 Development of flow net for dam on less permeable foundation (Example 5). (a) Construct flow net for dam, assuming foundation is completely impervious. (b) Extend equipotential lines into foundation without adjusting lines of net in dam. (c) Adjust equipotentials and flow lines until a balanced flow net is obtained.

the other part is completely impervious (Fig. 4.7a). The equipotentials are then extended down into the less permeable part (Fig. 4.7b), and by trial and error a balanced flow net is developed (Fig. 4.7c). When developing and using flow nets of this kind, fundamental relationships previously discussed should be kept in mind. In the completed flow net in Figure 4.7c one flow channel in the foundation conducts only one-tenth as much seepage as one flow channel in the more permeable embankment. For this reason the flow lines in the foundation are shown as dashed lines because they do not enclose full flow channels. The embankment in Figure 4.7 accommodates nearly one full flow channel; however, the first full flow channel (solid line) dips slightly into the foundation. Seepage beneath this flow line adds only about 0.32 of a full flow channel, thus raising the total value of n_f to 1.32. The number of equipotential

drops in this flow net is 7 and the shape factor is $1.32/7$ or 0.19 . Through the embankment alone (Fig. 4.7a) the shape factor would be approximately $0.9/7$, or 0.13 ; hence the relatively deep foundation increases the water losses only about 50%.

In constructing a flow net for a section of the type represented by this example, it is helpful to think in terms of the water-conducting capabilities of soils of different permeabilities. Thus, if a foundation has lower permeability than an embankment, as in this example, a greater thickness of the lower permeability soil is needed to conduct the same amount of water. Because most of the flow is through the embankment, the position of the equipotential lines in the embankment is influenced only slightly by the flow through the foundation.

Example 6 Earth Dam on a Highly Permeable Foundation. Figure 4.8 illustrates the same physical cross section used in Figure 4.7, but here the foundation is 10 times more permeable than the embankment. In this example the general shape of the net is controlled more by the dam.

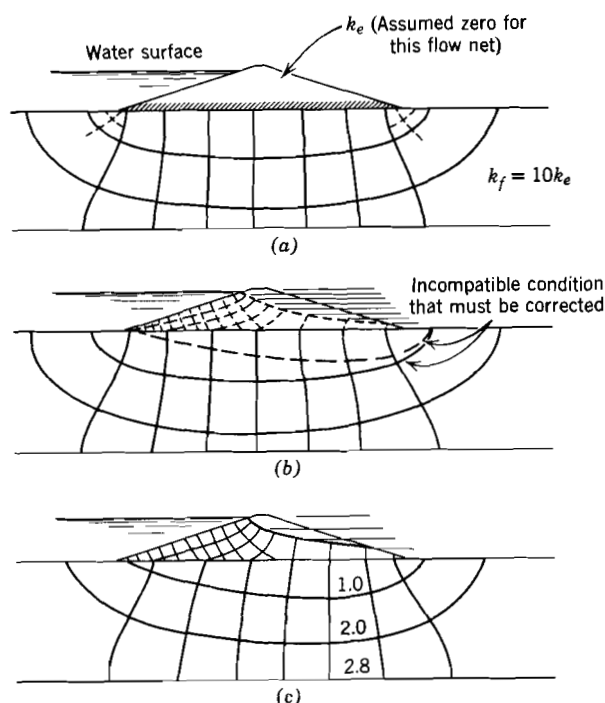


FIG. 4.8 Development of flow net for dam on more permeable foundation (Example 6). (a) Construct flow net assuming dam is completely impervious. (b) Extend equipotentials up into dam locating initial position of saturation line in dam. (c) Adjust equipotentials and flow lines until a balanced flow net is obtained.

A good starting procedure is to temporarily that the embankment (Fig. 4.8a) is for a *confined flow* system (phreatic line) is known. This trial net has been drawn (Fig. 4.8b) and the equipotential lines are then adjusted until a flow net is obtained. The differences between the permeabilities of the embankment and the foundation are then extended into the correct number of equipotential lines. In Figure 4.4a, horizontal head lines are drawn through the dam and each equipotential line is a phreatic line. (See also Figs. 3.6 and 3.7.)

In developing flow nets of this type, the position of the phreatic line is a function of the permeability of the flow net. This type of flow net is used at the start of the construction. At the start of the construction, the flow pattern can save much work. Initial approximations of the phreatic line should never be permitted to overshadow the final result.

4.7 EXAMPLES OF COMPLEX FLOW NETS

When the principles illustrated in the previous sections are applied to complex, interesting and useful flow net problems, the results are often cross sections. Two examples of these are shown in Figures 4.9 and 4.10. When large areas are involved, as in Figure 4.9, considerable work may be needed to construct a flow net properly. The flow net in Figure 4.9 is constructed on a cross-section of a dam. The dimensions have been reduced by a factor of 100. The flow net is drawn on the natural section of the dam.

Figure 4.10 gives two flow nets. The first is a study of seepage through a reservoir on the left from a dam. The dam is a layer of low permeability material to a depth of about 50 ft. The dam is a 50- to 60-ft thick sand layer. The dam is a permeable gravelly material. The

A good starting procedure is to draw a flow net for the foundation, assuming temporarily that the embankment is completely impervious. Such a net (Fig. 4.8a) is for a *confined flow system* in which the upper saturation line (the phreatic line) is known. This trial net is the type illustrated in Figure 4.3. After this trial net has been drawn (Fig. 4.8a) the equipotential lines are extended up into the embankment (Fig. 4.8b). The lines in both dam and foundation are then adjusted until a flow net (Fig. 4.8c) compatible with the assumed differences between the permeabilities of the two soil units is obtained. Before extending the flow net into the embankment, the total head h should be divided into the correct number of parts (eight in this example) to conform to the number of equipotential drops in the trial flow net in the foundation. As in Figure 4.4a, horizontal *head* lines are drawn across the downstream part of the dam and each equipotential line must intersect the correct head line at the phreatic line. (See also Figs. 3.6 and 4.4c.)

In developing flow nets of the type illustrated in Figures 4.7 and 4.8, the position of the phreatic line must be adjusted simultaneously with the refinement of the flow net. This type of flow net is the most difficult to construct. At the start of the construction, time taken to appraise the broad nature of the flow pattern can save much effort. Frequently shortcuts of the kind illustrated in Figures 4.7 and 4.8 help to obtain correct solutions with minimum work. Initial approximations of the type made in these two examples should never be permitted to overshadow the basic rules governing flow nets.

4.7 EXAMPLES OF COMPLEX FLOW NETS

When the principles illustrated in the preceding examples are carefully followed, interesting and useful flow nets can be developed for a wide variety of cross sections. Two examples of fairly complex flow nets are given here in Figures 4.9 and 4.10. When large differences in permeabilities exist in various zones as in Figure 4.9 considerable detail may be needed in one material but only a few lines may be needed in another when the basic checks are carried out properly. The flow net in Figure 4.9 is for an earth dam and foundation assumed to have horizontal permeabilities nine times vertical; hence the flow net is constructed on a *transformed section* (Fig. 4.9b) in which the horizontal dimensions have been reduced by the square root of the ratio of vertical-to-horizontal permeabilities, or $\sqrt{1/9} = 1/3$ (see sec. 3.3). the flow net is then redrawn on the natural section, as in Figure 4.9a.

Figure 4.10 gives two flow nets (constructed on *transformed sections* for $k_h = 25 k_v$) in a study of seepage beneath a dam built in a natural saddle separating a reservoir on the left from a lower valley on the right. Beneath a relatively thin layer of low permeability soil is a stratum of highly permeable sand and gravel to a depth of about 50 ft. Beneath the pervious sand and gravel layer is a 50- to 60- ft thick silty formation, which in turn is underlain by fairly permeable gravelly materials. Far to the right is a fairly steep slope (about 1

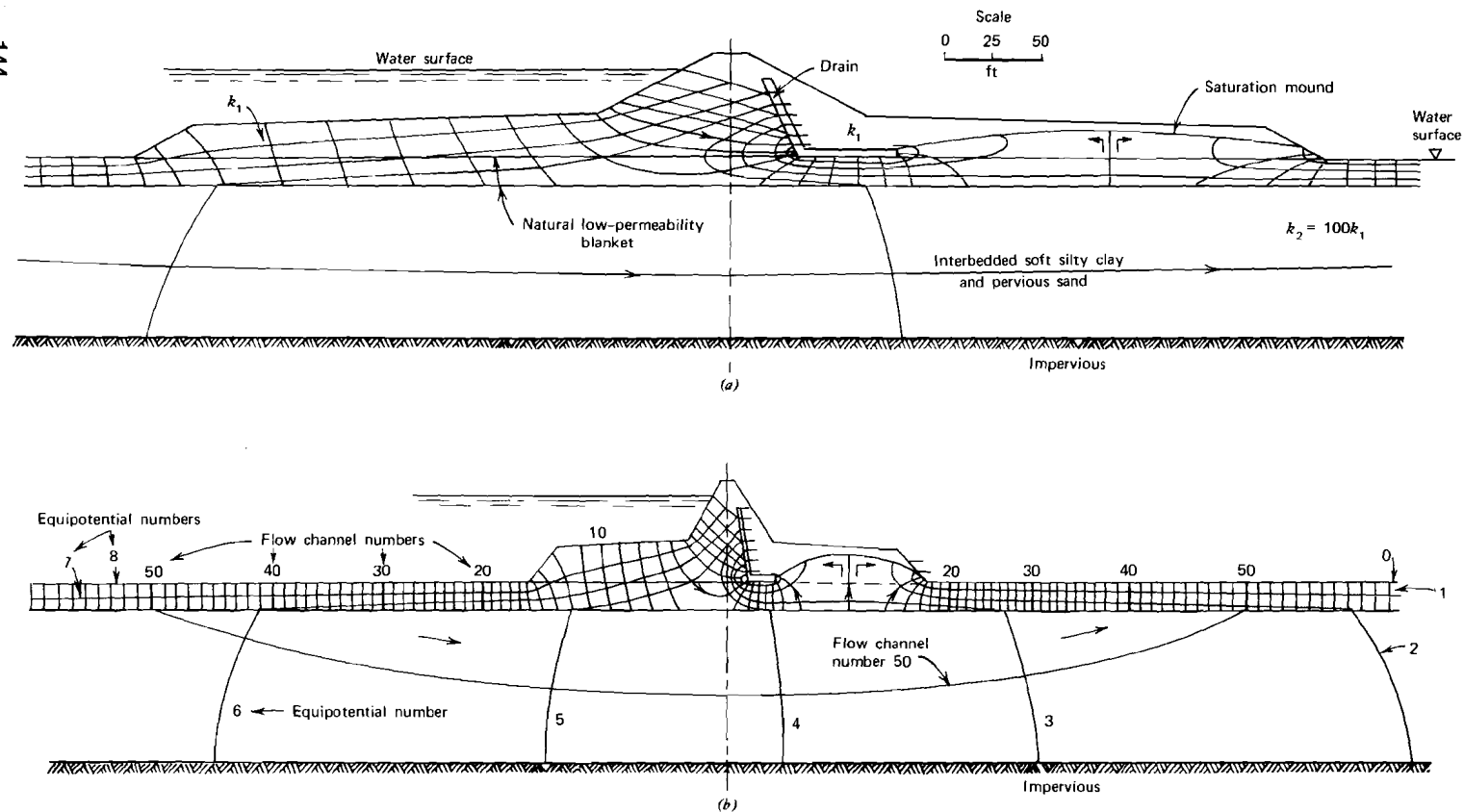
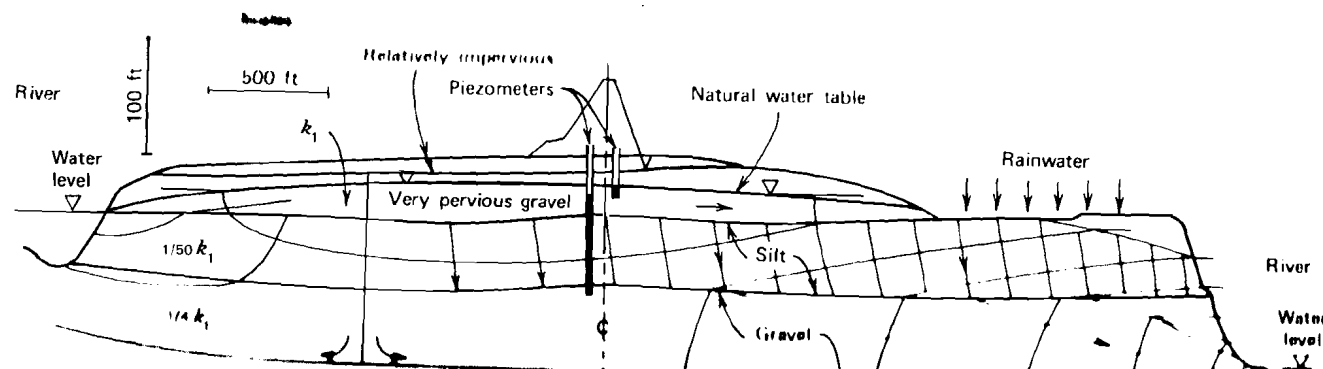


FIG. 4.9 Flow net study for an earth dam on a layered foundation. (a) True section ($k_h = 9k_v$). (b) Transformed section ($k_h = 9k_v$). (From *Embankment-Dam Engineering-Casagrande Volume*, Wiley, New York, 1973, chapter 3, "Seepage Control in Earth Dams," Harry R. Cedergren, p. 43.)



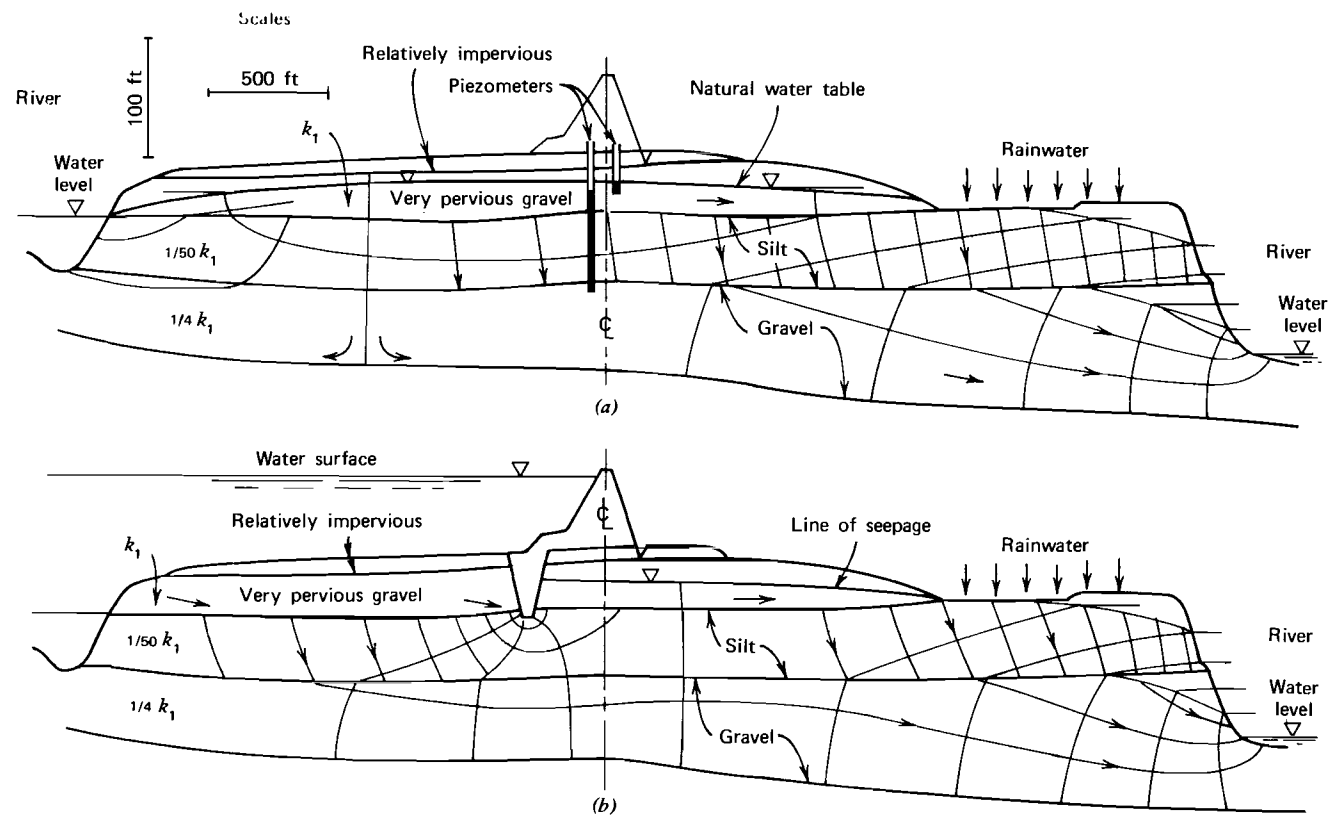


FIG. 4.10 Flow net study for a saddle dam on a layered foundation, transformed sections, $k_h = 25 k_v$. (a) Preexisting conditions, (b) Full reservoir, steady state. (From *Embankment Dam-Engineering-Casagrande Volume*, Wiley, New York, 1973, Chapter 3, "Seepage Control in Earth Dams," Harry R. Cedergren, p. 33.)

on 1) down to a river. On the transformed section this slope appears much steeper than it really is because horizontal dimensions have been shrunk to $1/5$ of natural (for the assumption that $k_h = 25k_v$, horizontal dimensions are multiplied by $\sqrt{k_h/k_v} = \sqrt{1/25} = 1/5$).

Preexisting piezometric levels had been observed at two or more depths at a number of locations at this site. These observations indicated the existence of downward hydraulic gradients, as smaller heads were measured in the bottom gravels than in the upper gravels at a given piezometer. Seepage evidently was escaping freely from the lower gravels where they emerged on the river bank at the right. Springs emerging from the silty formation relatively high above the river evidently were being fed by infiltrating rainwater in the general area being studied. Without such a supplemental source of water, the line of seepage could not have emerged at such high levels unless the soil cross section were very different from that shown.

The flow net in Figure 4.10a, which is for preexisting conditions, was constructed so that the equipotential lines were consistent with the observed piezometer readings. Flow nets constructed for a variety of design options indicated that the safest procedure would be to excavate a cutoff trench through the upper permeable gravels, into the silt layer, and tie the cutoff backfill into the impervious core of the dam, as shown by the flow net in Figure 4.10b, which indicates very safe conditions at the downstream toe of the dam. Details such as locations of filters are not shown in this drawing. This example is presented here as it illustrates the way flow nets should be adapted to the conditions existing at proposed sites for dams or other water-impounding structures.

In the construction of complex flow nets, such as the ones illustrated here, the most important single check is that provided by Eq. 3.11, namely that figures be elongated or shortened in proportion to the *relative* permeabilities of the various zones in a cross section. Many other extremely useful solutions to everyday problems can be obtained with some of the simplest kinds of flow net; hence one need not develop skill with the more complex types to be able to make good use of the flow net.

SUPPLEMENTAL READING

- Casagrande, A., "Seepage Through Dams," *Journal*, New England Water Works Association, June 1937, pp. 131-170; also publication No. 209, Graduate School of Engineering, Harvard University.
- Cedergren, Harry, R., "Use of Flow Net in Earth Dam and Levee Design," *Proceedings*, 2nd International Conference on Soil Mechanics and Foundation Engineering, Rotterdam (1948), Vol. 5, pp. 293-298.
- U.S. Department of the Army, Corps of Engineers (1986), "Seepage Analysis and Control for Dams," Engineering Manual EM 1110-2-1901, Office of the Chief of Engineers, Washington, D.C., 30 September 1986.

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