# ESTIMATING DYNAMIC O-D DEMANDS FOR A FREEWAY CORRIDOR USING LOOP DETECTOR DATA

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## ABSTRACT

Most studies of network operational improvements require the knowledge of origin-destination traffic demands. However, there are currently few O-D estimation techniques that have been demonstrated to be practical for the estimation of dynamic O-D traffic demands in a realistic network using field data. This paper describes the application of two previously proposed network O-D estimation models to a 35-km section of Highway 401 in Toronto, Canada. Typical of many field conditions, the true demands, the true routes, and the true route weights are all unknown. Furthermore, loop detector data, obtained from the COMPASS freeway traffic management system, are available for only 45% of the 35-km section of freeway. Fifteen-minute dynamic demands are estimated on the basis of FTMS loop detector data for the period of 5 AM to 11 AM. The results indicate that the estimated demands credibly reproduce the observed link flows ($r^2 > 0.92$), and that the two estimation models provide comparable estimates of the total number of vehicles and the average trip length. The relative strengths and weaknesses of the estimation techniques are discussed, and potential improvements are identified.

KEYWORDS: traffic demands, freeway operations, O-D estimation, simulation.

# INTRODUCTION

O-D demands are often considered to be independent, in the short term, of traffic control systems, and as such are often used to represent the independent input into the traffic engineering evaluation process. Unfortunately, trip-level data are typically not readily available and are usually very costly to obtain through direct measures. Many methods of estimating static, and to a lesser extent, dynamic O-D demands have been proposed over the past four decades. A substantial variety of approaches have been proposed including those that are based on observed link counts, license plate surveys, and socio-economic data. Summaries of the current state-of-the-art of O-D demand estimation are available in the literature (Cascetta and Nguyen, 1988; Willumsen, 1992; Cremer, 1992).

This paper describes the application of two recently proposed link-count based estimation models to a 35-kilometre section of multi-lane freeway in Toronto, Canada. Descriptions of the two estimation models, the LSE and LRE models, are available elsewhere in the literature (Van Aerde *et al*., 1993; Hellinga, 1994a)

Unlike hypothetical example networks, that are often used to illustrate the performance characteristics of many O-D estimation models, neither the true demands nor the routes or route weights are known for this freeway site. Furthermore, since the FTMS link flow data contain inconsistencies, neither node nor path flow continuity exist, with the result that no O-D demand solution exists that will exactly replicate the observed link flows. As these conditions are likely to be the norm rather than the exception for field applications, it is desirable to gain an understanding of how well the LSE and LRE models perform under these challenging conditions.

This paper is organised as follows. First, the two O-D estimation models are briefly described. Next, the 35-km section of Highway 401, for which demands are estimated, is described. The characteristics of the FTMS, that is operating on a portion of this section, and from which the necessary data are obtained, are presented. The process of extracting the data necessary for the estimation of time varying demands from the characteristics of the physical network and the available FTMS data is subsequently discussed. The extraction process involves abstracting the physical network into a series of nodes and directional links, determining a time series of observed link flows and link travel times, and identifying a time series of routes and route weights. Each of these three components is examined and described in turn. The demands estimated by the LSE and LRE models are evaluated by examining the aggregate link flow errors and by comparing estimated and observed flows for two individual links. Demand estimates are also compared to observed origin and destination zone productions and attractions. Lastly, conclusions are made about the performance of the dynamic LSE and LRE models.

## Description of the LSE and LRE Models

The LSE (least squared error) model is formulated on the premise that the best O-D estimate is the one that minimises the sum of the squared absolute deviations between estimated and observed link flows. This premise is also the basis of least-squares regression and has been used in various forms by other authors (Bell, 1991; Hendrickson and McNeil, 1984). The formulation of the LSE model is provided in Equation [1].

The LRE (least relative error) model is formulated on the premise that the best O-D estimate is the one that minimises the relative, instead of the absolute, link error (Equation [2]).

$$\min \quad E = \sum_a \left(V_a - V'_a\right)^2 \qquad [1]$$
$$\text{subject to} \quad T_{ij} \geq 0$$

$$\min \quad E = \sum_a \left[\ln\left(\frac{V_a}{V'_a}\right)\right]^2 \qquad [2]$$

where: $a$ = unique link identifier

$V_a$ = estimated flow on link $a$ (vph)

$$V_a = \sum_{ij} \left(T_{ij} P_{ij}^a\right)$$

$V'_a$ = observed flow on link $a$ (vph)

$P_{aij}$ = probability that demand between $i$ and $j$ will use link $a$

$T_{ij}$ = demand between origin $i$ and destination $j$ (vph)

## Description of the freeway site

The freeway site studied consists of a 35-km section of Highway 401 in Metropolitan Toronto, Canada (Figure 1). Carrying an average annual daily traffic (AADT) of approximately 320 000 vehicles and having a cross-section consisting of up to 16 directional lanes, this section of Highway 401 is the most heavily travelled section of freeway within Canada and ranks among the busiest in the world.

One unique feature of this freeway is that each direction of travel is provided with an express and collector facility. The collector facility provides primary access from and to the major arterial roads and other freeways connecting to Highway 401. The express facility has limited access to and from the collector facility via high-speed transfer roadways, providing drivers with a number of binary routing choices. At the time of this study, the system of express/collector facilities extended from Neilson Road in the east to Islington Avenue in the west.

## Description of the freeway traffic management system

Early in 1991, the Ontario Ministry of Transportation (MTO) brought on-line, a freeway traffic management system on Highway 401 through Toronto entitled COMPASS (Korpal, 1992). This system initially encompassed 16 kilometres of express and collector freeway. The system incorporated induction loop detectors embedded in the roadway approximately every 600 meters. This spacing provided detector coverage for each unique roadway segment and for all freeway access and egress roadways.

The loop detectors are connected via fibre optic cables to the operations control centre

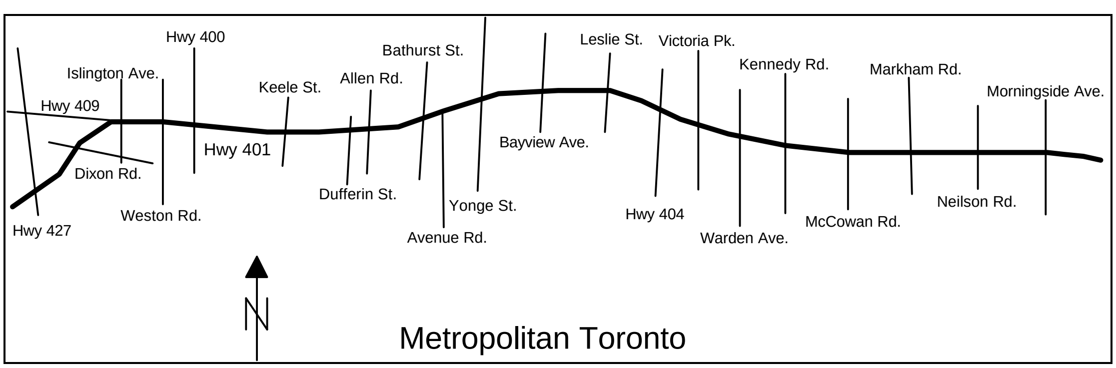


Figure 1: Highway 401 Network

located on the north side of Highway 401 at Keele Street. Initially, the loop detector stations were operational from Yonge Street, in the middle of the study area, west to the end of the study area. The detection system was expanded eastward to Warden Avenue, but this section was not operational during the course of this study.

## ACQUISITION OF DATA NECESSARY FOR O-D ESTIMATION

### Network representation

For this study, the Eastbound direction of Highway 401 was examined from Highway 427 in the west to Morningside Avenue in the east. This portion encompasses the entire express/collector facility existing during the summer of 1992. This section of Highway 401 includes 22 interchanges with connecting arterial roads and other freeways.

For the purposes of this study, this section of Highway 401 was abstracted into 239 directed links and 225 nodes. Links were defined such that a single link existed for each FTMS detector station. Link boundaries were defined such that detector stations were positioned near the midpoint of the link. This permitted the spot FTMS speed measurements to be used to estimate the link travel times.

### Link flows

The COMPASS FTMS detector stations measure and transmit volume, as well as occupancy and speed data, to the central computer every 20 seconds. Twenty second detector data for the two 24-hour periods of May 1, 1991 and June 8, 1992 were obtained from the MTO. An initial examination was carried out on the data from May 1. As reported by Hellinga and Van Aerde (1994a) this examination revealed several potential sources of errors arising from the manner in which the data are stored.

Significant variations over time were observed to exist in the 20-second volume data. To reduce this variation within the data, to reduce the storage requirements, and to provide a practical time slice duration over which time varying demands could be estimated, all data were aggregated into 15-minute average values.

### Route determination

Since the O-D demands are unknown, the traditional approach of using traffic assignment techniques for determining appropriate paths is not possible. For the estimation of O-D demands for the Highway 401 network, neither the utilised routes nor the O-D demands are known.

Since the true routes and route weights are unknown, some estimate of these quantities must be made. The simplest routing assumption is to route all traffic along the single shortest path. This approach is clearly unrealistic, as virtually all traffic demand would either use the collector lanes or the express lanes, exclusively. A more realistic approach is to assume that the two alternative paths, that are available, are both utilised in some proportion. It was assumed for the first path (express route), that drivers will attempt to access the express lanes at the earliest opportunity, and exit the express lanes at the latest possible point. For the second path (collector route), the collector lanes are utilised for the entire trip. In accepting these two paths as the only route alternatives, it is assumed that drivers do not arbitrarily alternate between express and collector lanes without having a distinct preference for either one facility or the other.

Though the routes have been identified, the proportion of demand using each route must still be determined. To simplify the problem, it was assumed that the proportion of demand utilising each route remained constant through both time and space for each direction. Thus, the proportion of demand choosing the express route is the same at the diversion point at Dixon Rd. during the a.m. period as it is at the transfer roadway at Markham Road during the p.m. period.

A previous simulation modelling study of Highway 401 determined, that for these same routing assumptions, the most appropriate split is for 80% of the demand to use the express route and 20% to use the collector route (Hellinga and Van Aerde, 1994a). This same split is assumed to be appropriate for this analysis. As noted by Hellinga and Van Aerde (1994b), this assumption is limiting since it is possible that this split changes over time of day as well as over distance. However, it is adequate for the purposes of illustrating the effectiveness of the LSE and LRE models when applied to field data.

<u>Link travel times</u>

The COMPASS FTMS detector stations measure and transmit volume, occupancy, and in the case of dual loop stations, speed data, to the central computer every 20 seconds. Single loop stations cannot measure speed directly, so for each 20 second interval, an average vehicle length that is computed from some nearby dual loop station, is used to compute an estimate of speed. Approximately 30% of the 175 detector stations that were operational in 1992 were dual loop stations.

An examination of the 20-second FTMS detector data revealed that the accuracy of the speeds estimated by single loop stations was, at that time, rather poor, particularly under low occupancy conditions. Two factors contributed to these inaccuracies:

1. The first is the inaccuracies in the recording of measured station occupancy. At that time, lane occupancy was measured in the field as a real value, but was truncated into integer format before being transmitted to the central computer. This truncated lane occupancy was used to compute the average station occupancy, which was also truncated into integer format before being stored on tape. The impact of this method is best illustrated using a simple example.

   Consider a three-lane section of freeway from which lane occupancy data are recorded. As illustrated in Table 1, the effect of dual truncation can lead to recorded station occupancies of zero, while in reality the occupancy was 1.6%. The amount of error introduced by this method of recording the station occupancy becomes less significant as the occupancy increases.

Table 1: Truncation effect on recorded station occupancies

| Lane Occupancy (%) | | Station Occupancy (%) | |
|---|---|---|---|
| True | Recorded | True | Recorded |
| 0.9<br>1.9<br>1.9 | 0<br>1<br>1 | (0.9+1.9+1.9)/3<br>= 1.57 | (0+1+1)/3<br>= 0 |

2. The second factor contributing to speed estimate inaccuracies is the need to have an estimate of average vehicle length for each single loop station. In 1992 each single loop station received an estimate of average vehicle length, every 20 seconds, from a nearby dual loop station. Unfortunately, the average vehicle length at two detector

stations, during a 20-second interval, is not strongly correlated, particularly at low occupancies. If a single truck is recorded during a 20-second interval at a dual loop station, then the average 20-second vehicle length at this station might be 20 meters. However, unless a single truck also happens to traverse, during the same time period, the single loop station to which the average vehicle length is being provided, the estimate of average vehicle length will be quite different from the actual average vehicle length. Under heavy flow conditions, more vehicles contribute to the computed average and the probability is lower that average vehicle lengths between stations will be very different.

Dual loop stations, which can directly measure speed, are considered to provide more reliable speed data. However, these data may also contain significant systematic error. In November 1992, it was discovered that, due to incorrect loop spacing, at least three dual loop detector stations in the westbound express lanes were reporting speed data with significant error[1]. Since that time, this error has been rectified; however, the data used in this study predate this time and contain these errors.

The existence of COMPASS on Highway 401 provides an efficient means of obtaining spot speeds and estimating link travel times. However, a previous study (Hellinga and Van Aerde, 1994c) has shown that the accuracy of travel times estimated from spot speeds is typically rather poor. Despite the poor level of correlation found in this study, no other link travel time data were available, and since some estimate of link travel time is required in order to estimate time-varying demands, the FTMS travel times were utilised.

As indicated earlier, FTMS data were only available for approximately 16 of the total 35-km network. For those regions of the network for which FTMS data were not available, link travel times were assumed to be the free speed travel time. This assumption is not accurate, unless link flows are quite small. A better approach may have been to calibrate a macroscopic speed-flow relationship, and estimate speeds based on the estimated flows. However, it is not possible to determine from flow alone, whether a link is operating in the uncongested or congested regions of the speed-flow relationship.

Potential sources of error within the estimation of link travel times must be evaluated in relation to the discretized time slice duration, and the average trip duration. If the time slice duration is much shorter than the average trip duration, then even small errors in the estimation of the link travel times can result in the association of a flow resulting from a particular demand to the incorrect time slice. When the time slice duration is longer, then larger link travel time errors can be tolerated before flows are associated with incorrect time slices.

Based on the data gathered during a floating car study (Hellinga and Van Aerde, 1994c), it was concluded that except for trips made during the Friday p.m. peak period, trips that traverse the entire study section, are not longer than 34 minutes in duration. Since the duration of the observation time slice, for the estimation of the time-varying O-D demands, is 15 minutes, the longest trips will be completed in approximately two time slices. Furthermore, these trip durations are representative of trips that traverse the entire 35 km section of freeway. It would be expected that most trips do not traverse the entire network and, as a result, would also have a shorter duration. However, the true trip

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[1]Personal communication with D. Tsui of the FTMS Section of the Ontario Ministry of Transportation. September 1993.

length distribution is not known, with the result that it is not possible to verify this expectation using field data. It will be shown, however, that both the LSE and the LRE models predict O-D demands that result in an average trip length of approximately 10-km, suggesting that many trips will be able to be completed within a single 15 minute time slice. Due to the short trip duration, relative to the time slice length, it is possible that the errors within the estimates of link travel times may ultimately have relatively little impact on the estimated O-D demands. It would be expected that these link travel time inaccuracies would have a more significant impact if the discretized time slice were to be made shorter, to say 5 minutes. An examination of these impacts is outside of the scope of this thesis.

## DETERMINATION OF TIME VARYING DEMANDS

The previous sections of this paper have described the physical network characteristics, the quantity and quality of the available FTMS link flow data, and route and route weight assumptions. Before examining any results, it is necessary to describe the parameter settings of the O-D estimation process.

FTMS link flow data, reflecting traffic conditions on Highway 401 between 5 a.m. and 11 a.m. on June 8, 1992 were utilised. These data were aggregated into 24 periods, each of a 15-minute duration. Neither path, nor node continuity of flow existed. Observed flows were available for only 68 of the 239 links within the network. To provide a constraint to the model for the links for which observed flows did not exist, the link's capacity was considered to be the *observed* flow whenever the estimated flow exceeded the link's capacity.

Since no prior information was available describing the magnitude or structure of the true O-D demand, the seed demand was defined to consist of a constant value in all inter-zonal O-D cells. To limit the amount of computing time required, a maximum of 30 iterations of the iterative solution algorithm was carried out.

### Evaluation of estimated demands with respect to aggregated observed link flows

A primary measure of the effectiveness of an O-D estimation method, is an aggregate measure of the link flow errors. One such convenient measure is the normalised link flow error, $\overline{E}^{n}$, in which a measure of the link flow error is represented as a proportion of the mean observed flow (Equation [3]). A perfect correlation between observed and estimated flows results in a normalised link flow error of 0.0.

$$\overline{E}^{n} = \frac{1}{\overline{V}'}\sqrt{\sum_{a}\left(V_a - V_a'\right)^2 \frac{1}{N}} \qquad [3]$$

where: $a$ = unique link identifier
$V_a$ = estimated flow on link $a$ (vph)
$V'_a$ = observed flow on link $a$ (vph)
$\overline{V}'$ = average observed link flow (vph)
$N$ = total number of links over which summation computed

After 30 iterations, the demand estimated by the LSE model resulted in a normalised link flow error of 13.9% of the average observed link flow. The average observed link flow, computed from 1705 link flow observations, was found to be equal to 2755 vph, while the average estimated link flow, computed over the same links, was equal to 2766 vph.

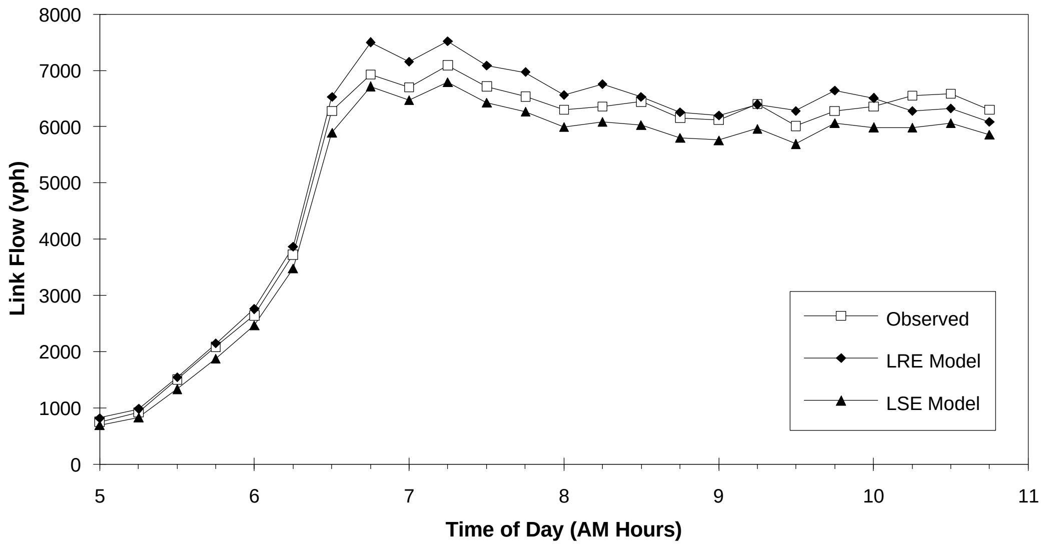


Figure 2: Comparison of observed and estimated flows by time of day
(Link 12: Express lanes between Hwy 409 and Hwy 400)

Similar results were obtained for the LRE model. The final normalised link flow error computed for the LRE model was 16.35%. The average estimated link flow, computed for only those links for which observed data were also available, was equal to 2724 vph.

Given the previously discussed errors in the flow data, the lack of observed data for a large portion of the system, the questionable reliability of the link travel time data, and the limiting assumptions made in selecting the routes, the obtained level of accuracy of the estimated link flows appears quite high.

Evaluation of estimated demands with respect to individual observed link flows

The previous section examined the performance of the LSE and LRE models by analysing the link flow error aggregated across all links and all time slices. It is also of interest to examine the correlation between observed and estimated link flows by time of day for several individual links. Since observed link flow data are available for 68 of the 239 links in the network, it is not practical to individually examine all 68 links. Rather, results are illustrated for a single link, link 12, located in the express lanes between the Highway 409 on-ramp and the off-ramp to Highway 400.

Figure 2, which illustrates the observed and estimated link flows by time of day for link 12, indicates that both the LSE and the LRE models predict flows that closely follow the trends in the observed data. Naturally, it is not possible to make any general conclusions based on these data, as it could be argued that this link is not representative of the entire system.

In order to make more general conclusions, the correlation between observed and estimated link flows can be determined for all of the 1705 link flow observations that are available. The computed correlation coefficient between the observed and estimated link flows is 0.9871 for the LSE model and 0.9821 for the LRE model. In both cases, the high level of correlation indicates that there is a strong association between the estimated

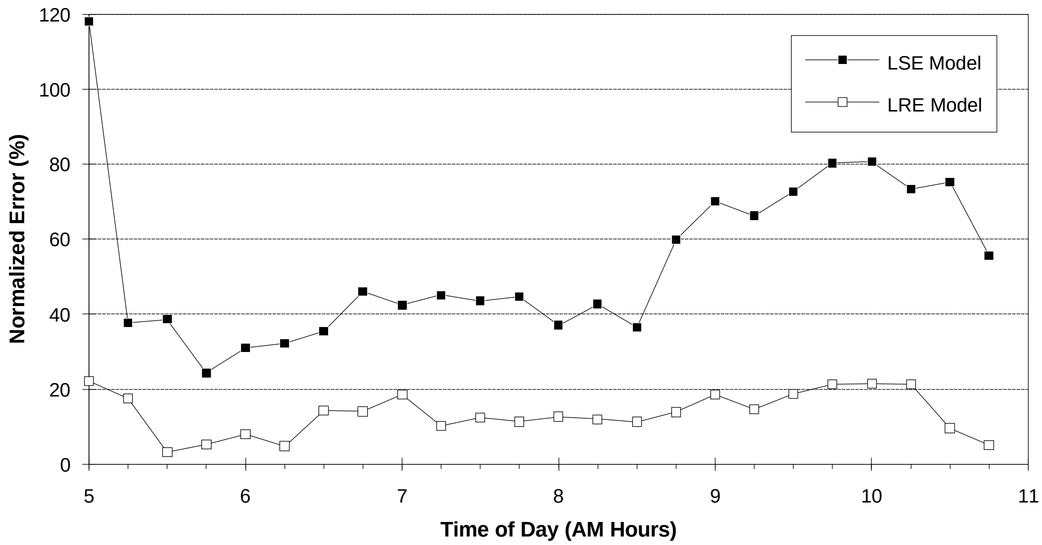


Figure 3: RMS error between estimated and observed total origin productions presented as a percent of the average observed total

and observed link flows. However, these values must be considered with some caution, as the observed data are not all strictly independent.

Evaluation of estimated demands with respect to observed origin productions and destination attractions

The measures of performance examined thus far have been limited to comparisons between estimated and observed link flows. The actual goal, however, is to estimate O-D demands that closely replicate the true demands. Since the true demands are unknown, it is not possible to make any direct comparisons between the estimated and true demands. It is possible, however, to observe the total number of origin productions and the total number of destination attractions. These values represent the flows observed on the links connected to each origin and to each destination zone. In this manner, comparisons can be made between observed and estimated row and column totals of the demand matrices.

Each interchange on the freeway was considered as both an origin and a destination zone. Based on this definition, it was possible for two or more links to be associated with each origin or destination zone. This occurs when two on-ramps, one carrying southbound flow, and the other carrying northbound flow, exist within the same junction. In these cases, the flows on these links were aggregated to form the total origin and destination flows.

It has been noted earlier that observed link flows are available for only 68 of the 239 links within the network. Observed data were not available for many of the defined origin and destination zones. In order to carry out a comparison, observed flows were required for all of the links that were associated with a zone. As a result, comparisons could only be made for four origin zones (Highway 409, Islington Ave., Weston Rd., and Yonge St.) and six destination zones (Dixon Rd., Islington Ave., Weston Rd., Keele St., Dufferin Ave., and Avenue Rd.). For each of these zones, the RMS error between the observed and estimated flows was computed as a proportion of the total observed flow.

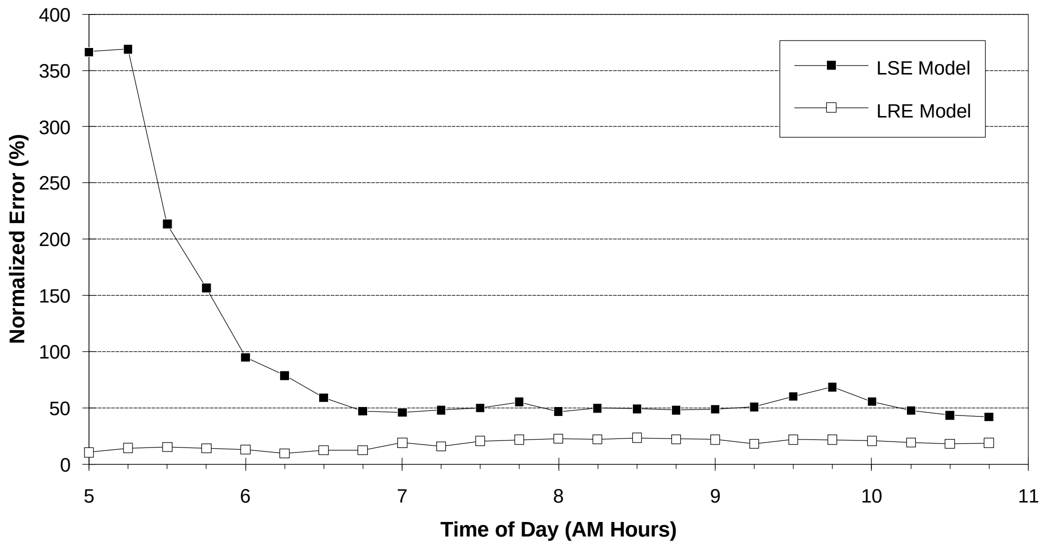


Figure 4: RMS error between estimated and observed total destination attractions presented as a percent of the average observed total

Figures 3 and 4 illustrate the results as a function of time of day for origins and destinations, respectively.

Several observations can be made on the basis of Figures 3 and 4. First, the LRE model provides much better estimates of the zonal flows than does the LSE model. Second, the quality of the LRE model estimates have much smaller variations across time of day, than do the LSE model estimates. This is particularly evident in Figure 4. This poor performance by the LSE model seems contradictory to the other performance measures examined thus far that indicated that the LSE model performed marginally better than the LRE model.

To provide some understanding for this apparent contradiction, one must first review the objective functions of the two models. The LSE model attempts to minimise the squared absolute link flow error aggregated across all links and all time slices. Since only the absolute error is considered, the algorithm first contends with links having large absolute error. Negative flows are not permitted, so the links with a large absolute error will tend to be those links that have large observed flows. Conversely, links with low observed flows are not likely to have a significant impact on the estimated O-D. Only 30 of the 68 links, for which observed data are available, are ramp links. The remaining 38 links are either express or collector links, having significantly higher flows than the ramp, or zonal, links. Thus, the LSE model places more weight on replicating the higher flow mainline links than the lower flow zonal links.

Consider Figures 5 and 6, which depict the correlation between the observed destination flows and those estimated by the LSE model and the LRE model, respectively. It is immediately apparent that there exists a much higher correlation between the LRE model estimates and the observed destination flows ($r = 0.95$), than between the LSE model estimates and the observed destination flows ($r = 0.75$). The LSE model tends to over-estimate the destination flows, particularly at low flows. This trend is consistent with the earlier observation that the LSE model tends to place more weight on links having large

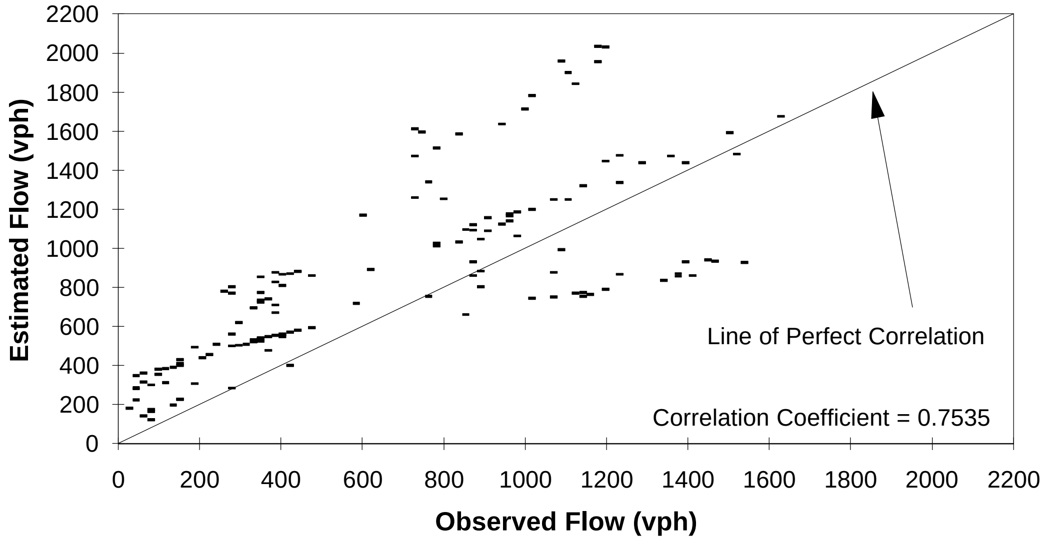


Figure 5: Correlation between observed total destination zone flows and those estimated by the LSE model

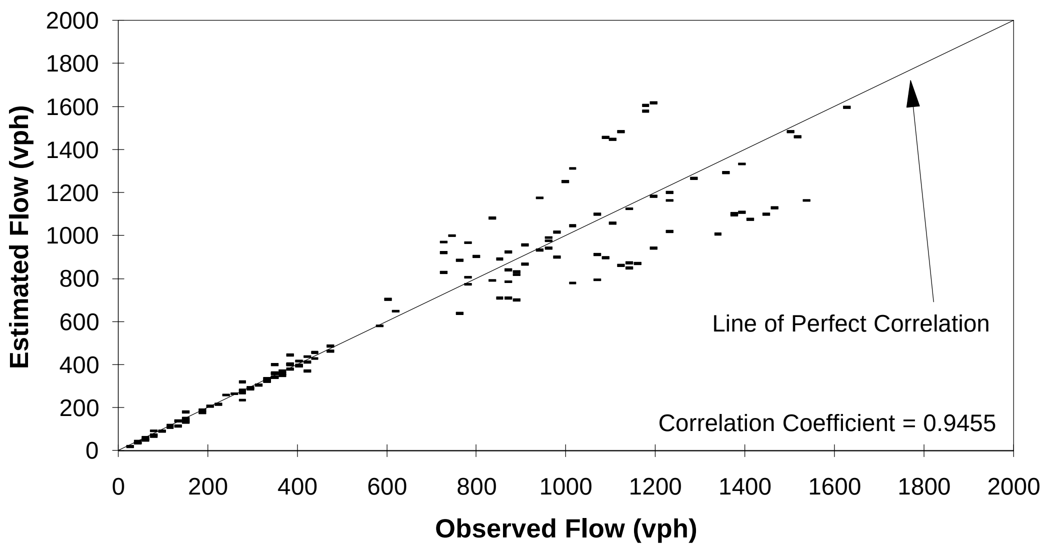


Figure 6: Correlation between observed total destination zone flows and those estimated by the LRE model

observed flows. Since flows cannot be negative, errors on links with low flow tend to result from an over-estimation of the flows.

Since the LRE model considers the link flow error relative to the observed flow, it would be expected that the flows estimated by the LRE model would have an approximately consistent relative error. Figure 6 indicates that there is a high degree of correlation between the observed destination flows and the LRE model estimates. Again, this is consistent with the model formulation, as a link flow error of 10 on a link with an observed flow of 100 contributes equally to the objective function as a link error of 1000 for a link with an observed flow of 10,000.

Further characteristics of the estimated O-D demands

In addition to the performance measures examined thus far, the estimation of the O-D demand permits the average trip length and the total number of trips to be computed. These characteristics cannot be computed without either the knowledge of the O-D demands, or alternatively, several assumptions, such as the average number of links traversed by a vehicle during a trip. The estimation of these characteristics provides for an opportunity to perform a reality check on the estimated O-D. Operators of FTMS and traffic engineers employed by departments of transportation often have some notion of the total number of trips that are made within a network during a specified time period. They may also have some estimate of the average trip length. A favourable comparison of these two quantities against the values derived from the estimated demands serves to place more confidence within the estimated demand.

The average trip length was derived from the estimated O-D demands and the known routes. The demand estimated by the LSE model implied an average trip length of 9.9 km, while the demand estimated by the LRE model implied an average trip length of 9.7 km. It is interesting to note that both models estimate approximately the same average trip length. Unfortunately, since no average trip length data were available for the Highway 401 network, it was not possible to compare the model estimates against field data.

The total number of trips represented by the O-D demand estimated by the LSE and LRE model could also be computed. The total number of trip departures during the six hour period from 5 a.m. to 11 a.m. on the eastbound direction of Highway 401 was estimated to be 160 400 trips by the LSE model and 171 000 by the LRE model. Again, no data were available for Highway 401 against which these estimates could be compared.

## CONCLUSIONS AND RECOMMENDATIONS

Highway 401 is a challenging urban freeway corridor to model. The problem of estimating static or dynamic demands are complicated by the existence of significant binary route choices due to the presence of the parallel express and collector facilities, and by the availability of FTMS data for only 45% of the entire system. Furthermore, there existed substantial systematic and random errors within the existing flow and speed data. All of these conditions produced an environment that was not ideal for estimating O-D demands, but was perhaps typical.

The choice of aggregation time slice duration (15 minutes) may be too long in view of the fact that few trips required more than 35 minutes to complete. However, a shorter time slice duration would significantly increase the impact of the errors within the travel time estimates.

Despite limited data, and the presence of systematic and non-systematic errors, both the LSE and LRE models were able to estimate a time-varying O-D demand. Based on the available measures of performance, both models were able to estimate a time-varying demand that successfully reflected the observed link flows.

It is recommended that dynamic O-D demands be re-estimated when the FTMS coverage has been extended to the full system. Furthermore, as the MTO has been made aware of the systematic errors within the FTMS data collection process, efforts have been made to eliminate, or at least reduce, the impacts of these errors. As such, more recently collected data should be of higher quality.


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## REFERENCES


Bell, Michael G.H. (1991) *The estimation of origin-destination matrices by constrained generalized least squares*. Transportation Research Vol. 25B No. 1. pp. 13-22.

Cascetta, Ennio and Nguyen, Sang. (1988) *A unified framework for estimating or updating origin/ destination matrices from traffic counts.* Transportation Research Vol. 22B No. 6. pp. 437-455.

Cremer, M. (1992) Origin-destination matrix: Dynamic estimation. *Concise Encyclopedia of Traffic and Transportation Systems*. Edited by Markos Papageorgiou. Pergamon Press. pp. 310-315.

Hellinga, B. (1994a) *Estimating Origin - Destination Demands from Link and Probe Counts*. Ph.D. Dissertation, Queen's University, Kingston, Canada.

Hellinga, B. (1994b) *QueensOD: User's Guide to model version 1.2c.* Transportation Systems Research Group, Queen's University, Kingston, Canada.

Hellinga, B., and Van Aerde, M. (1994a) *An Overview of a Simulation Study of the Highway 401 Freeway Traffic Management System*. Canadian Journal of Civil Engineering Vol. (21) pp. 439 - 454.

Hellinga, B. and Van Aerde, M. (1994b). *A Simulation Study of the Highway 401 FTMS*. Final report submitted to the Ontario Ministry of Transportation.

Hellinga, B. and Van Aerde, M. (1994c). *A Speed-Delay Study of Highway 401 in Toronto, Ontario.* Compendium of Technical Papers (Vol. 2) of the CITE Annual Conference held in Windsor, Ontario. pp. 405-420.

Hendrickson, C. and McNeil, S. (1984) *Estimation of origin-destination matrices with constrained regression*. Transportation Research Record 976. pp. 25-32.

Korpal, P.R. March 1992. *Incident management: the key to successful traffic management in Toronto.* Institute of Transportation Engineers Journal. pp. 58-61.

Van Aerde, M., Hellinga, B., MacKinnon, G., and Wang, J. (1993) *QueensOD: A practical method for estimating time varying origin-destination demands for freeway corridors/networks*. Presented at the Transportation Research Board Annual Meeting in Washington D.C. January 1993.

Willumsen, L.G. (1992) Origin-destination matrix: Static estimation. *Concise Encyclopedia of Traffic and Transportation Systems*. Edited by Markos Papageorgiou. Pergamon Press. pp. 315-322.