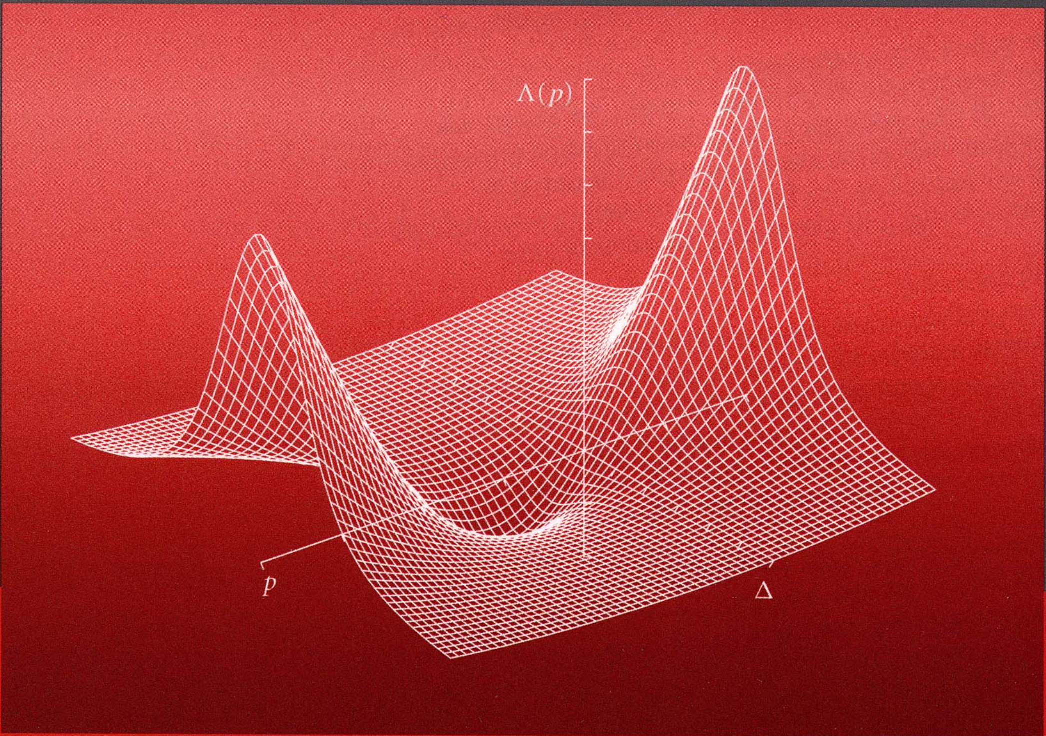
DYNAMIC STABILITY OF STRUCTURES



WEI-CHAU XIE

CAMBRIDGE

Dynamic Stability of Structures

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University of Waterloo

TO

My Family

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PART II
Dynamic Stability of Structures under Stochastic Loadings

Preface

Background and Scope of the Book

Investigation of the dynamic stability of elastic systems frequently leads to the study of the dynamic behaviour of the solutions derived from a parametrized family of differential equations. Examples of such systems include slender columns and thin plates under axial loading, or buildings, bridges, and aircraft structures under wind loading.

When the loadings are dynamic (either deterministic functions or random processes), the structures are then called parametrically excited. Parametric instability or resonance is characterized by exponential growth of the response amplitudes even in the presence of damping.

As a result, parametric resonance is more dangerous than ordinary resonance, in which the loading appears as the forcing term, rather than as a parameter, in the governing equations of motion.

The nature of the problems to be solved is characterized, in general, by the nature of the loading. When the loadings are deterministic periodic functions, the resulting governing equations of motion are of the Mathieu–Hill types; whereas when the excitations are random forces, the dynamics of structures is governed by stochastic differential equations. Hence, this book is divided into two parts, i.e. Part I: dynamic stability of structures under deterministic loadings (Chapters 2–4) and Part II: dynamic stability of structures under stochastic loadings (Chapters 5–9).

It is the purpose of this book to present a systematic introduction to the theory of parametric stability of structures under both deterministic and stochastic loadings.

Chapter 1 presents a general introduction to the concept of stability, conservative systems, nonconservative systems, and gyroscopic systems. Equations of motion of several structural systems are derived. The dynamic stability of these systems is studied throughout the book.

The dynamic stability of linear differential equations with periodic coefficients, i.e. Mathieu–Hill equations and Mathieu equations, is studied in Chapter 2.

The method of averaging, developed by Bogoliubov and Mitropolski, is applied in Chapter 3 to obtain the stability regions of Mathieu equations, linear multiple degrees-of-freedom non-gyroscopic and gyroscopic systems. Subharmonic and combination resonances of these systems are investigated.

In Chapter 4, nonlinear systems under periodic excitations are studied. The effect of nonlinearity on the stability of steady-state solutions is determined. Examples of a column under axial harmonic load and snap-through of a shallow arch are used to illustrate the procedures of analysis.

The theory of random processes, stochastic calculus, stochastic differential equations, and various techniques for solving these equations, such as the method of stochastic averaging and Monte Carlo simulation schemes, are presented in Chapter 5. This Chapter lays the necessary theoretical foundation for the study of stochastic dynamic stability.

Almost-sure stability of systems under the excitation of non-white ergodic random processes is investigated in Chapter 6.

Moment stability of stochastic dynamical systems is presented in Chapter 7. Both first and second moment stability conditions of a second-order system under combined harmonic and stochastic excitation, and a coupled multiple degrees-of-freedom linear system under stochastic excitation, are determined to illustrate the approaches.

The modern theory of stochastic dynamic stability is founded on Lyapunov exponents and moment Lyapunov exponents, which are presented in Chapters 8 and 9, respectively. The concepts of both exponents are introduced and a variety of application problems are studied through the determination of these characteristic numbers using various methods and techniques. The almost-sure asymptotic stability of a stochastic dynamical system is characterized by the largest Lyapunov exponent; whereas the pth moment stability is determined by the pth moment Lyapunov exponent. Furthermore, the Lyapunov exponent and the moment Lyapunov exponent characterize how rapidly the response grows or decays sample-wise and moment-wise, respectively.

Since the largest Lyapunov exponent is equal to the derivative of the pth moment Lyapunov exponent at $p = 0$, the moment Lyapunov exponent is the ideal avenue and the ultimate characteristic number for the study of the dynamic stability of stochastic dynamical systems. Knowledge of the moment Lyapunov exponent gives the almost-sure asymptotic stability of a stochastic dynamical system through the Lyapunov exponent. If the system is almost-surely stable, the pth moment becomes unstable when p is greater than the stability index, which is the non-trivial zero of the moment Lyapunov exponent.

The book is primarily for engineering students and practitioners as the main audience. Readers with a good knowledge of advanced calculus, linear algebra, probability, differential equations, engineering mechanics, and structural dynamics, which can be acquired in a relevant undergraduate program, should be able to follow the book. Of course, a certain degree of mathematical sophistication is helpful. The book is presented in a style that can be studied by an engineer with suitable background without sacrificing mathematical rigour. For Chapters 2–4 and 6–9, the basic theory is first presented. Application problems are then formulated and solved, sometimes using more than one approach. The emphasis is on applications and various methods and techniques, both analytical and numerical, for solving engineering problems. Theory and application problems are presented as self-contained as possible. All important steps of analysis are provided to make the book suitable as a textbook and especially for self-study. This book is not intended to be a complete research monograph; a comprehensive survey of the research publications is therefore not provided.

Computer software packages for symbolic computations, such as *Maple*, are very useful in mathematical analysis. However, they cannot replace learning and thinking. It is important to develop analytical skills and proficiency through "hand" calculations, which will also help the development of insight into the problems and appreciation of the solution process. By providing *Maple* programs for some typical problems that can be solved efficiently using *Maple*, a balanced presentation is attempted so that the readers can not only run the *Maple* programs to solve the problems on hand but also learn the frequently used commands and techniques. It is advisable to use *Maple* mainly as a tool for verification and checking rather than relying on it to solve every problem and being lost often in pages of *Maple* output.

Part I of the book presents the classic theory of dynamic stability of structures under deterministic loadings. These materials are suitable for a one term (semester) graduate course. In fact, a large part of the materials of Part I and some sections of Chapters 1 and 5 are based on the lecture notes of a graduate course taught by Professor S. T. Ariaratnam at the University of Waterloo. A draft of this book was used in a one term graduate course at Waterloo, in which materials in Chapters 1 to 5, and many sections of Chapters 8 and 9 were covered.

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This book would not be possible without the unfailing love and support of my mother, who has always believed in me. I particularly thank my wife Cong-Rong and lovely daughters Victoria and Tiffany for their love, understanding, encouragement, and support.

I appreciate hearing your comments through email (xie@uwaterloo.ca) or regular correspondence.

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This book presents a systematic introduction to the theory of parametric stability of structures under both deterministic and stochastic loadings. A comprehensive range of theories are presented and various application problems are formulated and solved, often using more than one approach. Investigation of an elastic system's dynamic stability frequently leads to the study of dynamic behavior of the solutions of parametrically excited systems. Parametric instability or resonance is more dangerous than ordinary resonance as it is characterized by exponential growth of the response amplitudes even in the presence of damping. The emphasis in this book is on the applications and various analytical and numerical methods for solving engineering problems. The materials presented are as self-contained as possible, with all of the important steps of analysis provided in order to make the book suitable as a graduate-level textbook and especially for self-study.

WEI-CHAU XIE is a Professor in the Department of Civil Engineering at the University of Waterloo. He is the author of numerous articles on dynamic stability of structures, nonlinear dynamics, and stochastic mechanics. He is the recipient of the Teaching Excellence Award in recognition of his exemplary record as a teacher.

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