# CivE 375 Assignment 5 solutions

① a) $K = 10^{-7}$ cm/s $\quad \theta = 0.37$ $\quad$ assume leachate density is the same as water

groundwater flux: $q = K \frac{dh}{dz} \approx K \frac{\Delta h}{\Delta z} = 10^{-7} \times \frac{2.6}{-2.6} = -10^{-7}$ cm/s

porewater velocity: $v = \frac{q}{\theta} = 2.7 \times 10^{-7}$ cm/s

$= 8.41$ cm/yr

$t = \frac{d}{v} = \frac{2.6 \text{ m}}{8.41 \text{ cm/yr}} \cdot \frac{100 \text{ cm}}{\text{m}} = 30.9$ years.

b)

$q = 10^{-7} \cdot \frac{12.6}{-2.6} = -4.87 \times 10^{-7}$ cm/s.

$\therefore$ $t = 6.26$ years

$\%$ change $= \frac{30.9 - 6.26}{30.9} = 80\%$

c) $D = 0.095$ m²/yr

From page 6-33 of notes:

$$\frac{C}{C_0} = \frac{1}{2} \operatorname{erfc}\left[\frac{x - vt}{2\sqrt{Dt}}\right]$$ For 1-D advection/dispersion

i) 1% leachate concentration $\frac{C}{C_0} = 0.01$

$\therefore$ $0.02 = \operatorname{erfc}\left[\frac{x - vt}{2\sqrt{Dt}}\right]$ $\rightarrow$ From table on page 6-34 of notes, $0.02 \approx \operatorname{erfc}[1.6]$.

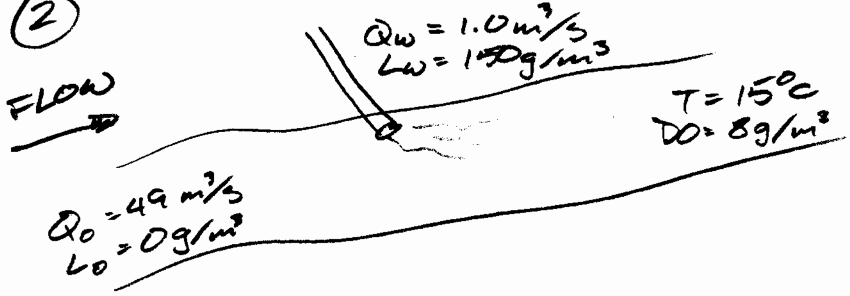
$\therefore$ $1.6 = \frac{x - vt}{2\sqrt{Dt}}$ $\rightarrow$ solve for $t$

$D = 0.095$ m²/y
$v = 2.7 \times 10^{-7}$ cm/s $= 0.085$ m/yr
$x = 2.6$ m.

} Watch your Units

$t = 4.07$ years. $\quad$ Repeat for $\frac{C}{C_0} = 0.1, 0.2$

(2)




$k_{20°C} = 0.23\,d^{-1}$

$K_2 = 0.15\,d^{-1}$

$\theta = 1.135 \quad 4°C < T < 20°C$

$K_{15} = K_{20}(1.135)^{(15-20)}$
$= 0.122\,d^{-1}$

Follow example 8.1 in text.

a) $Q_2 L_2 = Q_0 L_0 + Q_w L_w$

$$L_2 = \frac{(49)(0) + (1)(150)}{50} = 3\,g/m^3$$

From Appendix F in text. (oxygen-sag curve)

At 15°C, $C_s = 10.07$ (zero salinity)

$D_i = C_s - DO = 10.07 - 8 = 2.07\,g/m^3$

b) equation 8.22 of notes

$$\theta^*_H = \frac{1}{k_2 - k} \ln\left[\frac{k_2}{k}\left(1 - \frac{D_i(k_2 - k)}{k L_2}\right)\right]$$

$\theta^*_H$ - time to sag point

$\theta^*_H = 1.22$ days

(3) $k_2 = 0.58\,d^{-1}$ $\quad T = 15°C \quad Q = 10\,m^3/s \quad Q_w = 0.4\,m^3/s$

Find $BOD_5$-max while $DO \geq 4.0\,mg/L$

$k_{20} = 0.35\,d^{-1}$
$\therefore k_{15} = 0.19\,d^{-1}$

From Appendix F $\quad C_s = 10.07 \quad \therefore D_c = 6.07\,mg/L$

eq. 8.21 $\quad D_c = \frac{k}{k_2} L_i e^{-k\theta^*_H} = \frac{k}{k_2} L_c \quad \therefore L_c = 18.53\,mg/L$

Assume no initial oxygen deficit $\therefore D_i = 0$

$$\therefore \theta^*_H = \frac{1}{k_2 - k} \ln\left(\frac{k_2}{k}\right) = 2.86 \text{ days}$$

Find initial $BOD_{ult}$

$$L_i = L_c e^{\theta^*_H k} = 18.53 e^{(2.86)(0.19)} = 31.9\,mg/L$$

③ cont'd

From Mass Balance

$L_1 = 0$
$Q_1 = 10\ m^3/s$

$Q_w = 0.4\ m^3/s$
$L_w = ?$

$Q_2 = 10.4\ m^3/s$
$L_2 = L_i = 31.9\ mg/L$

$$Q_2 L_2 = Q_w L_w + \cancel{Q_1 L_1}^{\,0}$$

$$L_w = \frac{Q_2 L_2}{Q_w}$$

$$= 829.4\ mg/L$$

$$\therefore\ BOD_{5\,max} = L_w(1 - e^{-k_{20} \cdot 5})$$
$$= 829.4(1 - e^{-0.35(5)})$$
$$= 685.3\ mg/L$$

④

$Q_A = 0.28\ m^3/s$
$L_A = 45000\ kg/d$
$C_A = 0$

$Q_B = 0.57\ m^3/s$
$L_B = 34000\ kg/d$
$C_B = 0$

$Q_i = 36.8\ m^3/s$
$L_i = 0$
$C_i = C_s$
since:
$D_{O_2 i} = C_s - C_{O_2 i} = 0$

•A $\quad \bar{H}_{AB} = 4.6\ m$, $A_{AB} = 223\ m^2$, $d_{AB} = 32\ km$ •B $\quad \bar{H}_{BC} = 5.2\ m$, $A_{BC} = 279\ m^2$, $d_{BC} = 32\ km$ •C

$Q_{AB} = Q_i + Q_A$
$= 37.08\ m^3/s$

$Q_{BC} = Q_{AB} + Q_B$
$= 37.65\ m^3/s$

$k_{20} = 0.25\ d^{-1}$
$k_{r20} = 0.3\ d^{-1}$ $\quad \theta = 1.056$
$\therefore\ k_{23} = 0.294\ d^{-1}$
$k_{r23} = 0.353\ d^{-1}$

From Appendix F
$C_s$ @ $23°C = 8.56\ mg/L$

- convert $L_A$ and $L_B$ from kg/d to mg/L

$$L_A = \frac{45000\ kg}{d} \left| \frac{s}{0.28\ m^3} \right| \frac{m^3}{1000\ L} \left| \frac{10^6\ mg}{kg} \right| \frac{d}{86400\ s} = 1860\ mg/L$$

$$L_B = \frac{34000\ kg}{d} \left| \frac{s}{0.57\ m^3} \right. = 690\ mg/L$$

## Reach A-B

To solve this problem, we need to determine the remaining dissolved oxygen at the end of Reach A-B, using

$$D(t) = \frac{k L_{ABi}}{k_2 - k_r}(e^{-k_r t} - e^{-k_2 t}) + D_{ABi} e^{-k_2 t}$$ P. 7-11 of notes

we need:
- $L_{ABi}$ - initial $BOD_{ult}$ for reach A-B
- $k_2$ - coefficient of reaeration
- $D_{ABi}$ - initial D.O for reach A-B
- $t$ - time between point 'A' and 'B'

(4) cont'd To Find $k_2$, we need $D_{L23°C}$ and $U_{AB}$

P. 7-9 of notes $k_2 = \frac{294\sqrt{D_L U}}{\bar{H}^{3/2}}$ P. 7-15 of notes $D_{LT} = D_{L20}(1.037)^{T-20}$

$\therefore D_{L23} = (1.76\times10^{-4})(1.037)^3 = 1.963\times10^{-4}\ m^2/d$

$U_{AB} = \frac{Q_{AB}}{A_{AB}} = \frac{37.08\ m^3/s}{223\ m^2} = 0.17\ m/s$

$k_2 = \frac{294\sqrt{(1.963\times10^{-4})(0.17)}}{4.6^{3/2}} = 0.172\,d^{-1}$

Next we need the time for water to travel from A to B

$t_{AB} = \frac{d_{AB}}{U_{AB}} = \frac{32000\,m}{0.17\ m/s} = 2.18\ \text{days}.$

Use mass balance to find immediate D.O and BODult after loading at point source 'A'

$Q_{AB}C_{ABi} = \cancel{Q_A C_A}^{\to 0} + Q_i C_i \qquad D_i = C_s @ 23°C$

$C_{ABi} = \frac{Q_i D_i}{Q_{AB}} = \frac{(36.8)(8.56)}{37.08} = 8.50\ mg/L$

$Q_{AB}L_{ABi} = Q_A L_A + \cancel{Q_i L_i}^{\to 0} \qquad \therefore D_{ABi} = C_s - C_{ABi} = 0.06\ mg/L$

$L_{ABi} = \frac{Q_A L_A}{Q_{AB}} = \frac{(0.28)(1860)}{37.08} = 14.05\ mg/L$

Back to the Streeter-Phelps Oxygen sag model

$$D(2.18) = \frac{(0.294)(14.05)}{0.172-0.353}\left(e^{-0.353(2.18)} - e^{-0.172(2.18)}\right) + 0.06e^{-0.172(2.18)}$$

$D(2.18) = D_{AB} = 5.16\ mg/L$

$\therefore C_{AB} = C_s - D_{AB} = 3.40\ mg/L$

④ cont'd

Reach B.C.

Calculate a new $K_2$

$$U_{BC} = \frac{37.65\ m^3/s}{279\ m^2} = 0.135\ m/s \rightarrow t_{BC} = \frac{32000}{0.135} = 2.74\ days$$

$$K_2 = \frac{294\sqrt{(1.963\times10^{-4})(0.135)}}{5.2^{3/2}} = 0.128\ d^{-1}$$

Back to Mass balance

$$Q_{BC}C_{BC_i} = \cancel{Q_B C_B}^{\,0} + Q_{AB}C_{AB}$$

$$C_{BC_i} = \frac{37.08(3.40)}{37.65} = 3.35\ mg/L$$

$$\therefore D_{AB_i} = 8.56 - 3.35 = 5.21\ mg/L$$

$$Q_{BC}L_{BC_i} = Q_B L_B + Q_{AB}L_{AB}$$

where: $L_{AB} = L_{AB_i}e^{-kt_{AB}}$
$= (14.05)e^{-0.294(2.18)} = 7.40\ mg/L$

$$\rightarrow L_{BC_i} = \frac{(0.57)(690) + (37.08)(7.40)}{37.65} = 17.73\ mg/L$$

Plug values into Streeter-Phelps.

$$D(2.74) = \frac{(0.294)(17.73)}{0.128-0.353}\left(e^{-0.353(2.74)} - e^{-0.128(2.74)}\right) + 5.21e^{-0.128(2.74)}$$

$$D(2.74) = D_{BC} = 11.18\ mg/L > C_s.$$

$\therefore$ All Oxygen has been depleted

$C_{BC} = 0\ mg/L.$

system has become anaerobic

(5) Minimize $\text{cost} = 2.4\times10^4 R_A + 1.8\times10^4 R_B$

SEE Excel sheet

such the $C_{O_2,c} \geq 4.0\ \text{mg/L}$.

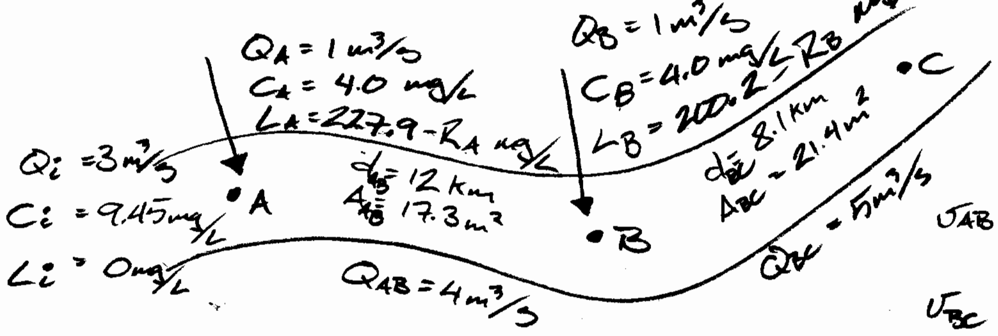
or $D_{O_2,max} = C_S - C_{O_2}$

$= 9.45 - 4.0$

$= 5.45\ \text{mg/L}$

$k_{18} = k_{20}\cdot(1.135)^{-2}$

$= 0.21\ d^{-1}$




Recall
$BOD_5 = L\left(1-e^{-5k}\right)$

$v_{AB} = \dfrac{4}{17.3} = 0.23\ \text{m/s} \rightarrow t_{AB} = 0.6\ \text{days}$

$v_{BC} = \dfrac{5}{21.4} = 0.23\ \text{m/s} \rightarrow t_{BC} = 0.4\ \text{days}$

Streeter - Phelps Equation...

**Reach AB**

$$C_{AB_i} = \frac{Q_i C_i + Q_A C_A}{Q_{AB}} = \frac{3(9.45) + 4.0}{4.0} = 8.1\ \text{mg/L} \quad \therefore D_{AB_i} = 1.4\ \text{mg/L}$$

$$L_{AB_i} = \frac{Q_i \cancel{L_i}^{\,0} + Q_A L_A}{Q_{AB}} = 0.25 L_A$$

$$D_{AB} = \frac{(0.21)(0.25)L_A}{0.43 - 0.3}\left[e^{-0.3(0.6)} - e^{-0.43(0.6)}\right] + 1.4e^{-.43(0.6)}$$

$$= 0.0253 L_A + 1.08\ \text{mg/L} \rightarrow C_{AB} = 8.37 + 0.0253 L_A$$

$$L_{AB} = L_{AB_i} e^{-0.21(0.6)} = 0.22 L_A$$

**Reach BC**

$$C_{BC_i} = \frac{Q_{AB} C_{AB} + Q_B C_B}{Q_{BC}} = \frac{4(8.37 + 0.0253 L_A) + 4}{5} = 7.5 + 0.02 L_A$$

$$D_{BC_i} = 1.95 + 0.02 L_A$$

$$L_{BC_i} = \frac{Q_{AB} L_{AB} + Q_B L_B}{Q_{BC}} = \frac{4\cdot 0.22 L_A + L_B}{5} = 0.176 L_A + 0.2 L_B$$

cont'd

(5) cont'd

Recall, we want $D_{BC} = D_{O_2,max} = 5.45$ mg/L

$$\therefore \quad 5.45 = \frac{0.21(0.176L_A + 0.2L_B)}{0.43 - 0.3}\left[e^{-0.3(0.4)} - e^{-0.43(0.4)}\right] + (1.95 + 0.02L_A)e^{-0.43(0.4)}$$

$$5.45 = (0.284L_A + 1.538L_B)(0.0449) + 1.642 + 0.0168L_A$$

$$3.808 = 0.0296L_A + 0.069L_B$$

Recall: $L_A = 227.9 - R_A$
$L_B = 200.2 - R_B$

$$3.808 = 20.56 - 0.0296R_A - 0.069R_B$$

simplify

$$\left.\begin{aligned} 16.8 &= 0.03R_A + 0.07R_B \\ \text{Min}\$ &= 2.4\times10^4 R_A + 1.8\times10^4 R_B \end{aligned}\right\} \text{2 equations, 2 unknowns.}$$

Restrictions $0 \le R_A \le 227.9$
$0 \le R_B \le 200.2$

set $R_B = \dfrac{16.8 - 0.03R_A}{0.07}$

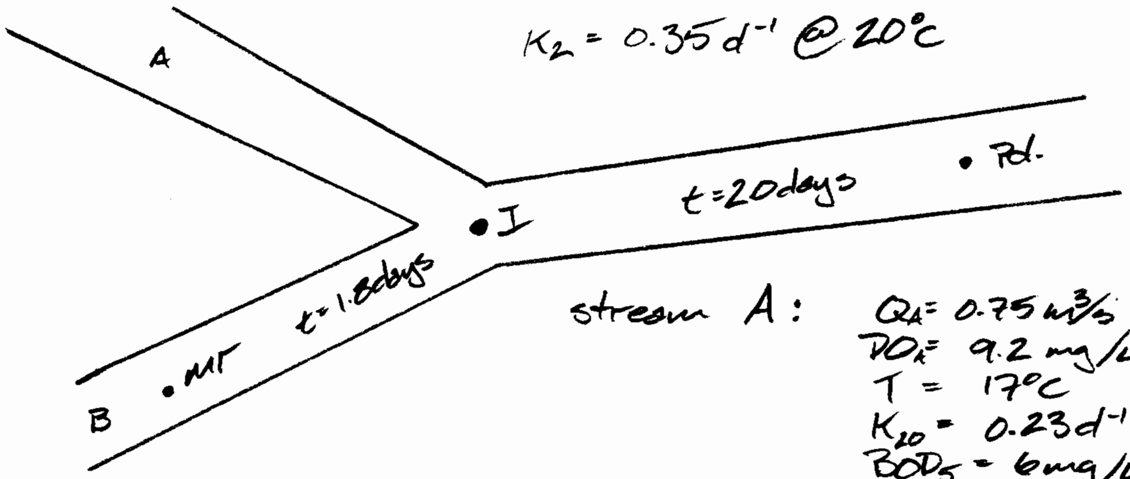
SEE EXCEL SHEET FOR SOLUTION

Min cost = \$5.9 million

where $R_A = 95$ mg/L
$R_B = 199.3$ mg/L

(6) Find $L_{Pd}$ and $DO_{Pd}$.

$K_2 = 0.35\,d^{-1}$ @ 20°C




stream A: $Q_A = 0.75\,m^3/s$
$DO_A = 9.2\,mg/L$
$T = 17°C$ $C_s = 9.65\,mg/L$
$K_{20} = 0.23\,d^{-1} \rightarrow K_{17} = 0.16\,d^{-1}$
$BOD_5 = 6\,mg/L$
$L_A = 10.9\,mg/L$

stream B: $Q_B = 3\,m^3/s$
$DO_B = 7.2\,mg/L$
$T = 16°C$ $C_s = 9.86\,mg/L$
$K_{20} = 0.23\,d^{-1} \rightarrow K_{17} = 0.14\,d^{-1}$
$BOD_5 = 4\,mg/L$
$L_B = 7.9\,mg/L$

After Intersection stream AB: $Q_{AB} = Q_A + Q_B + Q_{mr} = 4.75\,m^3/s$

meat rendering plant: $Q_{mr} = 1\,m^3/s$
$DO_{mr} = 0\,mg/L$
$T = 20°C$
$K_{20} = 0.23\,d^{-1}$
$\left.\begin{array}{l} BOD_5 = 300\,mg/L \\ L_{mr} = 439\,mg/L \end{array}\right\}$ untreated.

rate of reaeration = rate of deoxygenation

- For stream B, $BOD_5$ and DO. will not change before intersection point
- $K_2 (C_s - DO) = KL$

At Point mr.: $Q_B = 3\,m^3/s$ $Q_{mr} = 1\,m^3/s$ $Q_{Bmr} = 4\,m^3/s$

change in T $Q_{Bmr} T_{Bmr} = Q_i T_i + Q_{mr} T_{mr}$

$$T_{Bmr} = \frac{3(16) + 20}{4} = 17\,°C$$

$K_{17} = 0.16\,d^{-1}$

contd

(6) cont'd Find new $K_2$.

Recall $K_2 = \frac{294\sqrt{D_L U}}{\bar{H}^{3/2}}$ or $K_2 \propto \sqrt{D_L}$

P. 7-9 c.notes
P. 7-15

$$D_{L\,17} = 1.76\times10^{-4}(1.037)^{-3}$$
$$= 1.58\times10^{-4}\ m^2/day$$

$\therefore \frac{K_2 @ 20°C}{K_2 @ 17°C} = \sqrt{\frac{D_{L\,20}}{D_{L\,17}}} \rightarrow \frac{0.35}{K_2} = \sqrt{\frac{1.76}{1.58}} \rightarrow K_2 = 0.33$ @ 17°C

Find D.O after meat rendering plant.

$$Q_{Bmr} DO_{Bmr} = Q_i DO_i + \cancel{Q_{mr} DO_{mr}}^{0}$$

$$DO_{Bmr_i} = \frac{3(7.2)}{4} = 5.4\ mg/L \qquad C_{S_{17}} = 9.65\ mg/L$$

From $K_2(C_s - DO) = KL$ Find $L$

$$L = \frac{K_2}{K}(C_s - DO_{Bmr_i}) = 8.8\ mg/L \ @\ 17°C$$

- this is the initial $BOD_{UH}$ after the meat rendering plant, such that rate of reaeration = rate of deoxygenation $= L_{Amr_i}$

Also, at the intersection point, there will be no change in D.O and $BOD_{UH}$ for stream B:

$$L_{Bmr_i} = L_{Bmr} = 8.8\ mg/L$$
$$DO_{Bmr_i} = DO_{Bmr} = 5.4\ mg/L$$

At intersection Point: $Q_{AB} = 4.75\ m^3/s$. $T_{AB} = 17°C$
$K = 0.16\ d^{-1}$

$$DO_{AB_i} = \frac{Q_A DO_A + Q_{Bmr} DO_{Bmr}}{Q_{AB}} = \frac{(0.75)(9.2)+(4)(5.4)}{4.75} = 6\ mg/L$$

$$L_{AB_i} = \frac{Q_A L_A + Q_{Bmr} L_{Bmr}}{Q_{AB}} = \frac{(0.75)(10.9)+(4)(8.8)}{4.75} = 9.1\ mg/L$$

cont'd.

(6) cont'd ... Back to Streeter-Phelps ...

$$D(t) = \frac{K L_o}{K_2 - K_r}\left[e^{-K_r t} - e^{-K_2 t}\right] + D_o e^{-K_2 t}$$

Assume since we have no further information, $K_r = K$. See p 7-10 of note where $K_r \geq K$.

@ $T = 17°C$

$$D_o = (C_s - DO_{ABi}) = 9.65 - 6 = 3.65 \text{ mg/L}$$

$$D(2.0) = \frac{(0.16)(9.1)}{0.33 - 0.16}\left[e^{-0.16(2)} - e^{-0.33(2)}\right] + 3.65 e^{-0.33(2)}$$

$$= 3.68 \text{ mg/L} = C_s - DO_{pd}$$

$$\rightarrow DO_{pd} = 9.65 - 3.68 = 5.97 \text{ mg/L}$$

Find $BOD_{5,pd}$:

$$L_{pd} = L_{ABi} e^{-Kt} = 9.1 e^{-0.16(2)} = 6.61 \text{ mg/L}$$

$$BOD_5 = L_{pd}(1 - e^{-Kt})$$

$$= 6.61(1 - e^{-(0.16)(5)})$$

$$= 3.64 \text{ mg/L}$$