9. model for disinfection rate is given on page 565 of text

$$\frac{N_t}{N_0} = e^{-kt^m}$$

rearrange to linear form

$$\ln\left(\frac{N_t}{N_0}\right) = -kt^m$$

$$\ln\left(-\ln\left(\frac{N_t}{N_0}\right)\right) = \ln k + m \ln t$$

$\therefore$ a plot of $\ln\left(-\ln\left(\frac{N_t}{N_0}\right)\right)$ vs $\ln t$ has
slope of m, and
intercept of $\ln k$

using data set for free avail. chlor. $= 100\ g/m^3$

recognizing $\ln\left(-\ln\left(\frac{N_t}{N_0}\right)\right) = m \ln t + \ln k$

$$y = mx + b$$

$$m = \frac{\ln\left(-\ln\left(\frac{3e-4}{100}\right)\right) - \ln\left(-\ln\left(\frac{82}{100}\right)\right)}{\ln(16) - \ln(0)} = 1.5$$

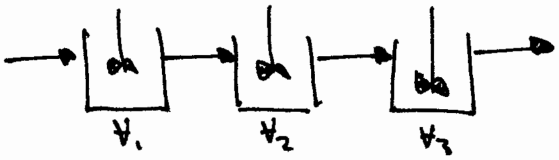
using $t = 16$ min to calculate k

$$\ln\left(-\ln\left(\frac{3e-4}{100}\right)\right) = (1.5)\ln(16) + \ln k$$

$$\underline{k = 0.199\ \text{min}^{1.5}}$$

q. 10 and 11 → see march 6 lecture notes (posted)

12.



$\forall_1 = \forall_2 = \forall_3$

- O_2 is put into system by mixer and taken out of the system by rxn with BOD

$r_{O_2\,transfer} = k_2 (C_s - C_{O_2})$

$r_{O_2\,total} = k_2 (C_s - C_{O_2}) - k L$

- mass balance for O_2 removal at CFSTR 1:

$$\begin{pmatrix}\text{rate of}\\ \text{mass accum}\end{pmatrix} = \begin{pmatrix}\text{mass}\\ \text{flux in}\end{pmatrix} - \begin{pmatrix}\text{mass}\\ \text{flux out}\end{pmatrix} \pm \begin{pmatrix}\text{source/}\\ \text{sink}\end{pmatrix}$$

$$\frac{d C_{O_2}}{dt} \forall = Q C_{O_2\,in} - Q C_{O_2\,out} + \left(k_2 (C_s - C_{O_2\,out}) - k L_{out}\right) \forall$$

- assume system is at steady-state

$$0 = Q C_{O_2\,in} - Q C_{O_2\,out} + \left(k_2 (C_s - C_{O_2\,out}) - k L_{out}\right) \forall$$

$$0 = \frac{C_{O_2\,in}}{t_o} - \frac{C_{O_2\,out}}{t_o} + k_2 (C_s - C_{O_2\,out}) - k L_{out}$$

$$C_{O_2\,out} = \frac{C_{O_2\,in} + k_2 C_s t_o - k L_{out} t_o}{(1 + k_2 t_o)}$$

* need to determine L

12 cont'd

- mass balance for L at CFSTR 1:

$$\frac{dL}{dt}V = Q L_{in} - Q L_{out} - k L_{out} V$$

- assume system is at steady state

$$0 = Q L_{in} - Q L_{out} - k L_{out} V$$

$$0 = \frac{L_{in}}{t_o} - \frac{L_{out}}{t_o} - k L_{out}$$

$$L_{out} = \frac{L_{in}}{1 + k t_o}$$

- plug in values and solve

$k = 10\ d^{-1}$, $k_2 = 80\ d^{-1}$, $C_s = 8\ g/m^3$, $L_{in} = 150\ g/m^3$

$t_o = 12\text{hr} \rightarrow 0.5\ d$ * units for t and k must be consistent

∴ to find O_2 in second tank:

- solve for L_1: $L_1 = \frac{L_{in}}{1+kt_o} = \frac{(150)}{1+(10)(0.5)} = 25\ g/m^3$

- solve for L_2: $L_2 = \frac{L_1}{1+kt_o} = \frac{(25)}{1+(10)(0.5)} = 4.17\ g/m^3$

- solve for $C_{O_2,1}$: $C_{O_2,1} = \frac{C_{O_2 in} + k_2 C_s t_o - k L_1 t_o}{(1+k_2 t_o)}$

$$C_{O_2,1} = \frac{(0) + (80)(8)(0.5) - (10)(25)(0.5)}{1+(80)(0.5)}$$

$$C_{O_2,1} = 4.76\ g/m^3$$

12 cont'd

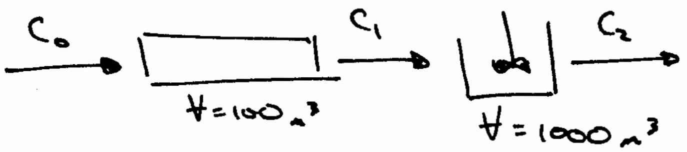
• solve for $C_{O_2,2}$:

$$C_{O_2,2} = \frac{C_{O_2,1} + k_2 C_S t_o - k L_2 t_o}{1 + k_2 t_o}$$

$$C_{O_2,2} = \frac{(4.76) + (80)(8)(0.5) - (10)(4.17)(0.5)}{1 + (80)(0.5)}$$

$$\underline{C_{O_2,2} = 7.41 \ g/m^3}$$

13. a) PFR → CFSTR



$Q = 0.05\ m^3/s$

$C_0 = 150\ g/m^3$

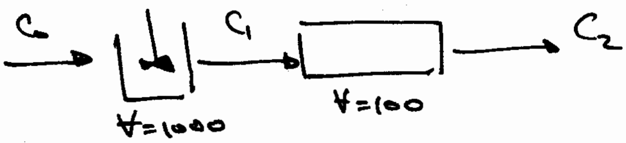
$$t_{o,PFR} = \frac{100}{0.05} = 2000\ s$$

$$t_{o,CFSTR} = \frac{1000}{0.05} = 20\,000\ s$$

PFR @ steady state $\quad C_1 = C_o e^{-kt_o} = (150)\, e^{-(10^{-5})(2000)} = \underline{\underline{147\ g/m^3}}$

CFSTR @ steady state $\quad C_2 = \dfrac{C_1}{1+kt} = \dfrac{(147)}{1+(10^{-5})(20000)} = \underline{\underline{122.5\ g/m^3}}$

b) CFSTR → PFR



CFSTR $\quad C_1 = \dfrac{C_o}{1+kt_o} = \dfrac{(150)}{1+(10^{-5})(20000)} = \underline{\underline{125\ g/m^3}}$

PFR $\quad C_2 = C_1 e^{-kt_o} = (125)\, e^{-(10^{-5})(2000)} = \underline{\underline{122.5\ g/m^3}}$

13 cont'd

· derive PFR and CFSTR equns for 2nd order rxn

PFR:
$$\frac{dC}{dt} = -v\frac{dC}{dx} - kC^2$$

@ steady state $\frac{dC}{dt} = 0$

$$0 = -\frac{dC}{dx} - kC^2$$

$$\frac{dC}{dx} = -\frac{k}{v}C^2$$

$$\int \frac{1}{C^2}\,dC = -\frac{k}{v}\int dx$$

$$-\frac{1}{C} = -\frac{k}{v}x + D$$

· determine value of D, constant of integration
for $C(x=0) = C_0 \rightarrow D = -\frac{1}{C_0}$

$$\frac{1}{C} = \frac{1}{C_0} + \frac{k}{v}x$$

$$\frac{1}{C(L)} = \frac{1}{C_0} + \frac{k}{v}L$$

$$\therefore \frac{1}{C(L)} = \frac{1}{C_0} + kt_0$$

note: $\frac{L}{v} = \frac{AL}{Av} = \frac{\forall}{Q} = t_0$

13 cont'd

CFSTR:

$$\forall \frac{\partial C}{\partial t} = Q C_0 - Q C - k C^2 \forall$$

@ steady state $\frac{\partial C}{\partial t} = 0$

$$0 = Q C_0 - Q C - k C^2 \forall$$

$$0 = \frac{C_0}{t_0} - \frac{C}{t_0} - k C^2$$

$$t_0 k C^2 + C - C_0 = 0$$

$$\therefore C = \frac{-1 \pm \sqrt{1 + 4 t_0 k C_0}}{2 t_0 k}$$

a) PFR → CFSTR

PFR $\frac{1}{C_1} = \frac{1}{C_0} + k t_0 = \frac{1}{(150)} + (10^{-5})(2000) \rightarrow \underline{C_1 = 37.5 \text{ g/m}^3}$

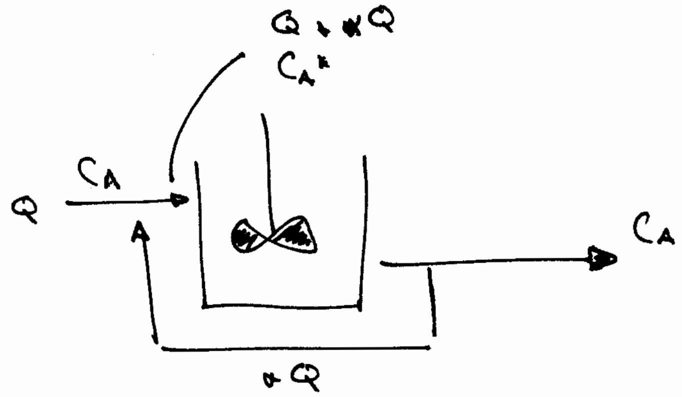
CFSTR $C_2 = \frac{-1 \pm \sqrt{1 + 4(20000)(10^{-5})(37.5)}}{2(20000)(10^{-5})} = \underline{11.4} \text{ g/m}^3$

b) CFSTR → PFR

CFSTR $C_2 = \frac{-1 \pm \sqrt{1 + 4(20000)(10^{-5})(150)}}{2(20000)(10^{-5})} = \underline{25 \text{ g/m}^3}$

PFR $\frac{1}{C_2} = \frac{1}{(25)} + (10^{-5})(2000) \rightarrow C_2 = \underline{16.7 \text{ g/m}^3}$

14.



- mass balance for CFSTR @ steady state

$$0 = (Q + \alpha Q) C_A^* - (Q + \alpha Q) C_A - r C_A$$

$$0 = Q C_{A_{in}} + \cancel{\alpha Q C_A} - Q C_A - \cancel{\alpha Q C_A} - r C_A$$

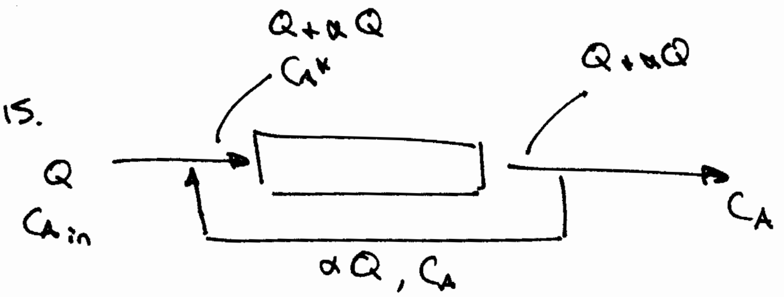
$$0 = Q C_{A_{in}} - Q C_A - r C_A$$

↳ from this point derivation of expression for C_A is same as pg 6-17.

∴ α has no effect for a steady state response

- if rxn order increases (e.g. C_A^2 vs C_A) then C_A at steady state decrease (see q. 13)

15.



$C_A = C_A^* \, e^{-kt_o}$ for a 1st order rxn @ steady state

note: $t_o = \dfrac{V}{Q + \alpha Q}$

$$C_A^* = \frac{Q\,C_{Ain} + \alpha Q\,C_A}{Q + \alpha Q}$$

$$C_A^* = \frac{C_{Ain} + \alpha C_A}{1 + \alpha}$$

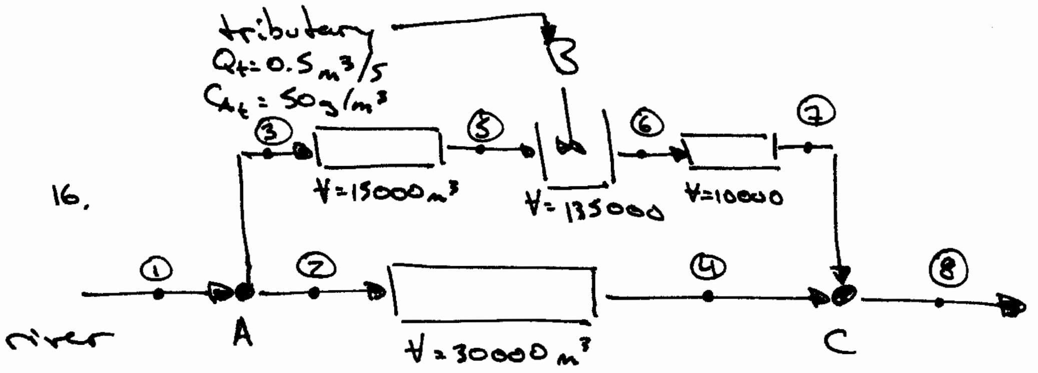
$$\therefore \; C_A = \left(\frac{C_{Ain} + \alpha C_A}{1 + \alpha}\right) e^{-kt_o}$$

$$C_A(1+\alpha) = (C_{Ain} + \alpha C_A)\, e^{-kt_o}$$

$$C_A = \frac{\dfrac{C_{Ain}}{1+\alpha}\, e^{-kt_o}}{1 - \left(\dfrac{\alpha}{1+\alpha}\right) e^{-kt_o}}$$

- unlike the CFSTR at steady state, α will effect the effluent conc.

16.



$Q_r = 4\ m^3/s$

$C_{Ar} = 100\ g/m^3$

$k = 0.5\ d^{-1}$

- calc. effulent from PFR - AC:

$$Q_{AC} = 0.3\,Q_r = 0.3(4) = 1.2\ m^3/s$$

$$t_{o,AC} = \frac{\forall}{Q} = \frac{(30000)}{(1.2)(86400)} = 0.289\ d$$ ← units need to be consistent with k

$C_2 = C_1$ ← splitting the flow does not change the conc.

$$C_4 = C_2\, e^{-k\, t_{o,AC}}$$

$$C_4 = (100)\, e^{-(0.5)(0.289)}$$

$$C_4 = 86.5\ g/m^3$$

- calc. effluent along A → B → C:

$C_3 = C_1$ ← splitting the flow does not change the conc.

$$Q_{AB} = 0.7\,Q_r = (0.7)(4) = 2.8\ m^3/s$$

$$t_{o,AB} = \frac{\forall}{Q} = \frac{(15000)}{(2.8)(86400)} = 0.062\ d$$

16. cont'd

$$C_5 = C_3 e^{-k\, t_{o,AB}}$$

$$C_5 = (100)\, e^{-(0.5)(0.062)}$$

$$C_5 = 96.9 \ g/m^3$$

- calc. C_6, derive mass balance expression for CFSTR for river and tributary inflows:

$$\forall_B \frac{dC_6}{dt} = Q_{AB} C_5 + Q_t C_t - Q_{BC} C_6 - k C_6 \forall_B$$

- assume steady-state condition

$$0 = Q_{AB} C_5 + Q_t C_t - Q_{BC} C_6 - k C_6 \forall_B$$

$$C_6 = \frac{Q_{AB} C_5 + Q_t C_t}{Q_{BC} + k \forall_B}$$, note: $Q_{BC} = Q_{AB} + Q_t = (2.8) + (0.5) = 3.3 \ m^3/s$

$$C_6 = \frac{(2.8)(96.9) + (0.5)(50)}{(3.3) + \left(0.5 \times \frac{1}{86400}\right)(135000)}$$

$$C_6 = 72.6 \ g/m^3$$

$$C_7 = C_6 e^{-k\, t_{o,BC}}$$, $t_{o,BC} = \frac{\forall_{BC}}{Q_{BC}} = \frac{(10000)}{(3.3)(86400)} = 0.035 \ d$

$$C_7 = (72.6)\, e^{-(0.5)(0.035)}$$

$$C_7 = 71.3 \ g/m^3$$

16 cont'd

- mass balance at point C
  - steady state, no accumulation $\therefore$ in = out

out = in
(to point 8) = (from point 4 and point 7)

$$Q_{total} C_8 = Q_{AC} C_4 + Q_{BC} C_7$$

$$C_8 = \frac{Q_{AC} C_4 + Q_{BC} C_7}{Q_{total}}, \quad Q_{total} = Q_{AC} + Q_{BC} = 1.2 + 3.3 = 4.5 \ m^3/s$$

$$C_8 = \frac{(1.2)(86.5) + (3.3)(71.3)}{(4.5)}$$

$$C_8 = \underline{75.4 \ g/m^3}$$