An Assumed Strain Blending Element Formulation for Linear Elastic Fracture Mechanics

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An Assumed Strain (AS) formulation for the blending elements of the eXtended Finite Element Method (XFEM) for Linear Elastic Fracture Mechanics (LEFM) problems is presented which shows improved accuracy and optimum convergence rates. The XFEM was introduced in the context of LEFM problems in two dimensions in [1]. Since its conception many papers have shown the improved accuracy of the XFEM over the standard FEM for singular fields, but until recently optimum convergence rates for the method have been elusive.

The primary source of error in the XFEM approximation was identified in [2] to reside in the blending elements, those elements where only a subset of nodes are enriched. As will be shown, a consequence of this error is that the traction free boundary conditions of the crack surfaces in the vicinity of the crack tip are poorly satisfied, which is a major assumption of most Stress Intensity Factor (SIF) calculations. This error is a result of the local Partition of Unity property of the XFEM which leads to parasitic terms in the approximation space of the blending elements.

Using the framework described in [3], we will develop an AS approximation for three node triangular blending elements for use with the branch function enrichment which will reduce the influence of the parasitic terms. The resulting approximation is superior of the accuracy of the energy norm, of the SIFs, and of the satisfaction of traction boundary conditions; in addition, it maintains an optimal convergence rate.

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