THEORETICAL SOIL MECHANICS

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NINTH PRINTING

London: CHAPMAN AND HALL, Limitea

JOHN WILEY AND SONS, INC.

NEW YORK

Section B

CONDITIONS FOR SHEAR FAILURE IN IDEAL SOILS

Chapter V

ARCHING IN IDEAL SOILS

18. Definitions. If one part of the support of a mass of soil yield while the remainder stays in place the soil adjoining the yielding part moves out of its original position between adjacent stationary masses of soil. The relative movement within the soil is opposed by a shearing resistance within the zone of contact between the yielding and the stationary masses. Since the shearing resistance tends to keep the yielding mass in its original position, it reduces the pressure on the yielding part of the support and increases the pressure on the adjoining stationary part. This transfer of pressure from a yielding mass of soil onto adjoining stationary parts is commonly called the *arching effect*, and the soil is said to *arch* over the yielding part of the support. Arching also takes place if one part of a yielding support moves out more than the adjoining parts.

Arching is one of the most universal phenomena encountered in soils both in the field and in the laboratory. Since arching is maintained solely by shearing stresses in the soil, it is no less permanent than any other state of stress in the soil which depends on the existence of shearing stresses, such as the state of stress beneath the footing of a column. For instance, if no permanent shearing stresses were possible in a sand, footings on sand would settle indefinitely. On the other hand, every external influence which causes a supplementary settlement of a footing or an additional outward movement of a retaining wall under unchanged static forces must also be expected to reduce the intensity of existing arching effects. Vibrations are the most important influence of this sort.

In the following article two typical cases will be investigated, viz., arching in an ideal sand due to the local yield of a horizontal support and arching in the sand adjoining a vertical support whose lower part yields in an outward direction.

19. State of stress in the zone of arching. The local yield of the horizontal support of a bed of sand shown in Figure 17*a* can be produced by gradually lowering a strip-shaped section *ab* of the support. Before the strip starts to yield, the vertical pressure per unit of area on the horizontal support is everywhere equal to the depth of the layer of sand

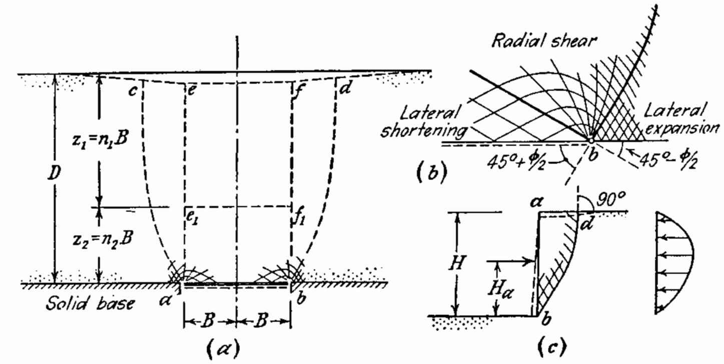

FIG. 17. Failure in cohesionless sand preceded by arching. (*a*) Failure caused by downward movement of a long narrow section of the base of a layer of sand; (*b*) enlarged detail of diagram (*a*); (*c*) shear failure in sand due to yield of lateral support by tilting about its upper edge.

times its unit weight. However, a lowering of the strip causes the sand located above the strip to follow. This movement is opposed by frictional resistance along the boundaries between the moving and the stationary mass of sand. As a consequence the total pressure on the yielding strip decreases by an amount equal to the vertical component of the shearing resistance which acts on the boundaries, and the total pressure on the adjoining stationary parts of the support increases by the same amount. In every point located immediately above the yielding strip the vertical principal stress decreases to a small fraction of what it was before the yield commenced. The total vertical pressure on the base of the layer of sand remains unchanged, because it is always equal to the weight of the sand. Therefore the decrease of the vertical pressure on the yielding strip must be associated with an increase of the vertical pressure on the adjoining parts of the rigid base, involving an abrupt increase of the intensity of the vertical pressure along the edges of the strip. This discontinuity requires the existence of a zone of radial shear comparable to that shown in Figure 15*a*. The radial shear is associated with a lateral expansion of the sand located within the high-pressure zone, on both sides of the yielding strip towards the low-pressure zone located above the strip. If the base of the layer of sand were perfectly smooth, the corresponding shear pattern should be similar to that indicated in Figure 17*a* and, on a larger scale, in Figure 17*b*.

As soon as the strip has yielded sufficiently in a downward direction, a shear failure occurs along two surfaces of sliding which rise from the outer boundaries of the strip to the surface of the sand. In the vicinity of the surface all the sand grains move vertically downward. This has been demonstrated repeatedly by time-exposure photographs. Such a movement is conceivable only if the surfaces of sliding intersect the horizontal surface of the sand at right angles. When the failure occurs a troughlike depression appears on the surface of the sand as indicated in Figure 17*a*. The slope of each side of the depression is greatest where it intersects the surface of sliding. The distance between these steepest parts of the trough can be measured. It has been found that it is always greater than the width of the yielding strip. Hence, the surfaces of sliding must have a shape similar to that indicated in Figure 17*a* by the lines *ac* and *bd*. The problem of deriving the equations of the surfaces of sliding *ac* and *bd* has not yet been solved. However, experiments (Völlmy 1937) suggest that the average slope angle of these surfaces decreases from almost 90° for low values of $D/2B$ to values approaching $45° + \phi/2$ for very high values of $D/2B$.

The vertical pressure on the lower part of the mass of sand located between the two surfaces of sliding, *ac* and *bd* in Figure 17*a*, is equal to the weight of the upper part reduced by the vertical component of the frictional resistance which acts on the adjoining surfaces of sliding. This transfer of part of the weight of the sand located above the yielding strip onto the adjoining masses of sand constitutes the arching effect.

The preceding reasoning can also be applied to the analysis of the arching effect produced in a mass of sand by the lateral yield of the lower part of a vertical support. In Figure 17*c* the lateral support is represented by *ab*. The surface of the sand is horizontal and the support yields by tilting around its upper edge. After the support has yielded sufficiently, a shear failure occurs in the sand along a surface of sliding *bd* which extends from the foot *b* of the support to the surface of the sand. The stationary position of the upper edge, *a*, of the lateral support prevents a lateral expansion of the upper part of the sliding wedge. Therefore the sand grains located in the upper part of the wedge can move only in a downward direction. Hence the surface of sliding intersects the horizontal surface of the sand at *d* at right angles. The corresponding subsidence of the surface of the sliding wedge is indicated in the figure by a dashed line.

The lateral expansion of the lower part of the sliding wedge is associated with a shortening in a vertical direction. The corresponding subsidence of the upper part of the wedge is opposed by the frictional resistance along the adjoining steep part of the surface of sliding. As a consequence the vertical pressure on the lower part of the wedge is smaller than the weight of the sand located above it. This phenomenon constitutes the arching effect in the sand behind yielding lateral supports whose upper part is stationary.

20. Theories of arching. Most of the existing theories of arching deal with the pressure of dry sand on yielding horizontal strips. They can be divided into three groups. The authors of the theories of the first group merely considered the conditions for the equilibrium of the sand which is located immediately above the loaded strip without attempting to investigate whether or not the results of the computations were compatible with the conditions for the equilibrium of the sand at a greater distance from the strip. The theories of the second group are based on the unjustified assumption that the entire mass of sand located above the yielding strip is in a state of plastic equilibrium.

In the theories of a third group it is assumed that the vertical sections *ae* and *bf* (Fig. 17*a*) through the outer edges of the yielding strip represent surfaces of sliding and that the pressure on the yielding strip is equal to the difference between the weight of the sand located above the strip and the full frictional resistance along the vertical sections (Cain 1916 and others). The real surfaces of sliding, *ac* and *bd* (Fig. 17*a*), are curved and at the surface of the sand their spacing is considerably greater than the width of the yielding strip. Hence the friction along the vertical sections *ae* and *bf* cannot be fully active. The error due to ignoring this fact is on the unsafe side.

The following comments are intended to inform the reader in a general way on the fundamental assumptions of the theories of the first two groups. Engesser (1882) replaced the sand located immediately above the yielding strip by an imaginary arch and computed the pressure on the strip on the basis of the conditions for the equilibrium of the arch. Bierbaumer (1913) compared the sand located immediately above the strip to the keystone in an arch. He assumed that the base of the keystone coincides with the surface of the strip, and that the sides of the keystone are plane and rise from the outer boundaries of the strip towards the center. The pressure on the strip is equal and opposite to the force required to maintain the keystone in its position. Caquot (1934) replaced the entire mass of sand located above the yielding strip by a system of arches. He assumed that the horizontal normal stress in the arches above the center line of the strip is equal to the corresponding vertical normal stress times the flow value N_ϕ, equation 7(4), and he computed the pressure on the strip on the basis of the conditions for the equilibrium of the arches. Völlmy (1937) replaced the curved surfaces of sliding *ac* and *bd* (Fig. 17*a*) by inclined plane surfaces and assumed that the normal stresses on these surfaces are identical with the normal stresses on similarly oriented sections through a semi-infinite mass of sand in an active Rankine state. The slope of the surfaces of sliding is chosen such that the corresponding pressure on the yielding strip is a maximum. According to the results of some of his investigations an increase of the angle of internal friction of the sand should cause an increase of the pressure on the yielding strip. According to all the other theories and to the existing test results an increase of the angle of internal friction has the opposite effect. Völlmy (1937) also investigated the pressure of the earth on rigid and on flexible culverts and compared the results of his analysis with those obtained by earlier investigators. However, under field conditions the pressure on yielding horizontal supports such as the roofs of culverts or of tunnels depends on many conditions other than those which have been considered so far in theoretical investigations.

All the theories cited above are in accordance with experience in that the pressure on a yielding, horizontal strip with a given width increases less rapidly than the weight of the mass of sand located above the strip and approaches asymptotically a finite value. However, the values furnished by different theories for the pressure on the strip are quite different. In order to find which of the theories deserves preference it would be necessary to investigate experimentally the state of stress above yielding strips and to compare the results with the basic assumptions of the theories. Up to this time no complete investigation of this type has been made, and the relative merit of the several theories is still unknown. The simplest theories are those in the third category which are based on the assumption that the surfaces of sliding are vertical. Fortunately the sources of error associated with this assumption are clearly visible. In spite of the errors the final results are fairly compatible with the existing experimental data. Therefore the following analysis will be based exclusively on the fundamental assumptions of the theories in this category. In connection with a scientific study of the subject Völlmy's publication should be consulted (Völlmy 1937).

If we assume that the surfaces of sliding are vertical as indicated by the lines *ae* and *bf* (Fig. 17*a*) the problem of computing the vertical pressure on the yielding strip becomes identical with the problem of computing the vertical pressure on the yielding bottom of prismatic bins.

For cohesionless materials this problem has been solved rigorously by Kötter (1899). It has also been solved with different degrees of approximation by other investigators. The simplest of the solutions is based on the assumption that the vertical pressure on any horizontal section through the fill is uniformly distributed (Janssen 1895, Koenen 1896). This assumption is incompatible with the state of stress on vertical sections through the soil, but the error due to this assumption is not so important that the assumption cannot be used as a basis for a rough estimate.

Figure 18*a* is a section through the space between two vertical surfaces of sliding. The shearing resistance of the earth is determined by the equation

$$s = c + \sigma \tan \phi$$

The unit weight of the soil is γ and the surface of the soil carries a uniform surcharge q per unit of area. The ratio between the horizontal

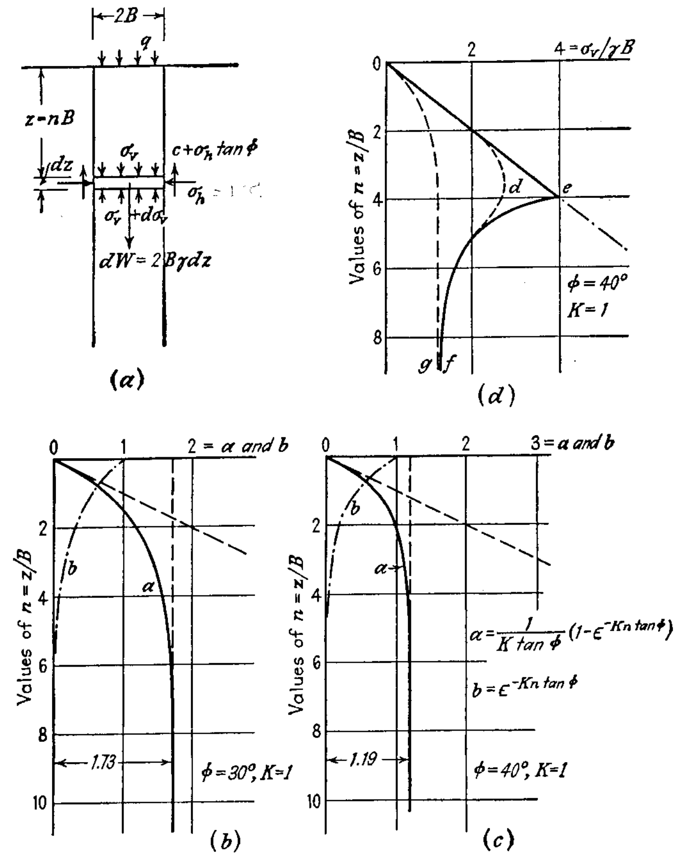

FIG. 18. (*a*) Diagram illustrating assumptions on which computation of pressure in sand between two vertical surfaces of sliding is based; (*c* and *d*) representations of the results of the computations.

and the vertical pressure is assumed to be equal to an empirical constant K at every point of the fill. The vertical stress on a horizontal section at any depth z below the surface is σ_v, and the corresponding normal stress on the vertical surface of sliding is

$$\sigma_h = K\sigma_v \quad [1]$$

The weight of the slice with a thickness dz at a depth z below the surface is $2B\gamma\, dz$ per unit of length perpendicular to the plane of the drawing. The slice is acted upon by the forces indicated in the figure. The condition that the sum of the vertical components which act on the slice must be equal to zero can be expressed by the equation

$$2B\gamma\, dz = 2B(\sigma_v + d\sigma_v) - 2B\sigma_v + 2c\, dz + 2K\sigma_v\, dz \tan\phi$$

or

$$\frac{d\sigma_v}{dz} = \gamma - \frac{c}{B} - K\sigma_v \frac{\tan\phi}{B}$$

and

$$\sigma_v = q \quad \text{for} \quad z = 0$$

Solving these equations we obtain

$$\sigma_v = \frac{B(\gamma - c/B)}{K \tan\phi}(1 - \epsilon^{-K\tan\phi\, z/B}) + q\epsilon^{-K\tan\phi\, z/B} \quad [2]$$

By substituting in this equation in succession the values $c = 0$ and $q = 0$, we obtain

$$c > 0 \quad q = 0 \quad \sigma_v = \frac{B(\gamma - c/B)}{K\tan\phi}(1 - \epsilon^{-K\tan\phi\, z/B}) \quad [3]$$

$$c = 0 \quad q > 0 \quad \sigma_v = \frac{B\gamma}{K\tan\phi}(1 - \epsilon^{-K\tan\phi\, z/B}) + q\epsilon^{-K\tan\phi\, z/B} \quad [4]$$

$$c = 0 \quad q = 0 \quad \sigma_v = \frac{B\gamma}{K\tan\phi}(1 - \epsilon^{-K\tan\phi\, z/B}) \quad [5]$$

If the shearing resistance in a bed of sand is fully active on the vertical sections ae and bf (Fig. 17a), the vertical pressure σ_v per unit of area of the yielding strip ab is determined by equation 5. Substituting in this equation

$$z = nB$$

we obtain

$$\sigma_v = \gamma a B \quad [6a]$$

wherein

$$a = \frac{1}{K\tan\phi}(1 - \epsilon^{-K\tan\phi\, z/B}) = \frac{1}{K\tan\phi}(1 - \epsilon^{-Kn\tan\phi}) \quad [6b]$$

For $z = \infty$ we obtain $a = 1/K\tan\phi$ and

$$\sigma_v = \sigma_{v\infty} = \frac{\gamma B}{K\tan\phi} \quad [7]$$

In Figure 18b the ordinates of the curve marked a represent the values of $n = z/B$ and the abscissas the corresponding values of a for $\phi = 30°$ and $K = 1$, or for $K\tan\phi = 0.58$. Figure 18c contains the same data for $\phi = 40°$ and $K = 1$ or for $K\tan\phi = 0.84$.

Experimental investigations regarding the state of stress in the sand located above a yielding strip (Terzaghi 1936e) have shown that the value K increases from about unity immediately above the center line of the yielding strip to a maximum of about 1.5 at an elevation of approximately $2B$ above the center line. At elevations of more than about $5B$ above the center line the lowering of the strip seems to have no effect at all on the state of stress in the sand. Hence we are obliged to assume that the shearing resistance of the sand is active only on the lower part of the vertical boundaries ae and bf of the prism of sand located above the yielding strip ab in Figure 17a. On this assumption the upper part of the prism acts like a surcharge q on the lower part and the pressure on the yielding strip is determined by equation 4. If $z_1 = n_1 B$ is the depth to which there are no shearing stresses on the vertical boundaries of the prism $abfe$ in Figure 17a the vertical pressure per unit of area of a horizontal section e_1f_1 through the prism at a depth z_1 below the surface is $q = \gamma z_1 = \gamma n_1 B$. Introducing this value and the value $z = z_2 = n_2 B$ into equation 4 we obtain

$$\sigma_v = \gamma B a_2 + \gamma B n_1 b_2 = \gamma B(a_2 + n_1 b_2) \quad [8a]$$

wherein

$$a_2 = \frac{1}{K\tan\phi}(1 - \epsilon^{-Kn_2\tan\phi}) \quad \text{and} \quad b_2 = \epsilon^{-Kn_2\tan\phi} \quad [8b]$$

For $n_2 = \infty$ the value a_2 becomes equal to

$$a_\infty = \frac{1}{K\tan\phi}$$

and the value b_2 equal to zero. The corresponding value of σ_v is

$$= \sigma_{v\infty}\gamma B a_\infty = \frac{\gamma B}{K\tan\phi}$$

which is equal to the value given by equation 7. In other words, the value $\sigma_{v\infty}$ is independent of the depth z_1 in Figure 17a.

The relation between n_2 and a_2 is identical with the relation between

n and a, represented by equation 6b and by the plain curves in Figures 18b and 18c. The relation between the values n and the corresponding values of

$$b = \epsilon^{-Kn \tan \phi}$$

is represented in Figures 18b and 18c by the dash-dotted curves marked b.

In order to illustrate by means of a numerical example the influence of the absence of shearing stresses on the upper part of the vertical sections ae and bf in Figure 17a we assume $\phi = 40°$, $K = 1$, and $n_1 = 4$. Between the surface and a depth $z_1 = n_1 B = 4B$ the vertical pressure on horizontal sections increases like a hydrostatic pressure in simple proportion to depth, as indicated in Figure 18d by the straight line oe. Below a depth z_1 the vertical pressure is determined by equations 8. It decreases with increasing depth, as shown by the curve ef and it approaches asymptotically the value $\sigma_{v\infty}$ (eq. 7).

The dashed line og in Figure 18d has been plotted on the assumption $n_1 = 0$. The abscissas of this curve are determined by equations 6. With increasing depth they also approach the value $\sigma_{v\infty}$ (eq. 7). The figure shows that the influence of the absence of arching in the upper layers of the bed of sand on the pressure σ_v on a yielding strip practically ceases to exist at a depth of more than about $8B$. Similar investigations for different values of ϕ and of n_1 led to the conclusion that the pressure on a yielding strip is almost independent of the state of stress which exists in the sand at an elevation of more than about $4B$ to $6B$ above the strip (two or three times the width of the strip).

If there is a gradual transition from full mobilization of the shearing resistance of the sand on the lower part of the vertical sections ae and bf in Figure 17a to a state of zero shearing stress on the upper part, the change of the vertical normal stress with depth should be such as indicated in Figure 18d by the line odf. This line is similar to the pressure curve obtained by measuring the stresses in the sand above the center line of a yielding strip (Terzaghi 1936e).

Less simple is the investigation of the effect of arching on the pressure of sand on a vertical support such as that shown in Figure 17c. The first attempt to investigate this effect was made on the simplifying assumption that the surface of sliding is plane (Terzaghi 1936c). According to the results of the investigation the arching in the sand behind a lateral support with a height H eliminates the hydrostatic pressure distribution and it increases the vertical distance H_a between the point of application of the lateral pressure and the lower edge of the support. The intensity of the arching effect and its influence on the value of the ratio H_a/H depends on the type of yield of the support. If

the support yields by tilting around its lower edge no arching occurs. The distribution of the earth pressure is hydrostatic and the ratio H_a/H is equal to one third. A yield by tilting around the upper edge is associated with a roughly parabolic pressure distribution and the point of application of the lateral pressure is located near midheight. Finally, if the support yields parallel to its original position, the point of application of the lateral pressure may be expected to descend gradually from an initial position close to midheight to a final position at the lower third point. The investigation gave a satisfactory general conception of the influence of the different factors involved, but, owing to the assumption that the surface of sliding is plane, failed to give information regarding the effect of arching on the intensity of the lateral pressure.

In order to obtain the missing information it was necessary to take the real shape of the surface of sliding into consideration. Since the upper edge of the lateral support does not yield, the surface of sliding must intersect the top surface of the backfill at right angles (see Art. 19).

Ohde investigated the influence of this condition on the intensity of the earth pressure on the assumption that the trace of the surface of sliding on a vertical plane is an arc of a circle which intersects the surface of the backfill at right angles (Ohde 1938). The corresponding lateral pressure and the location of the point of application of the lateral pressure have been computed for an ideal sand, with an angle of internal friction $\phi = 31°$, by three different methods.

In one of these, the location of the centroid of the pressure has been determined in such a manner that the stresses along the surface of sliding satisfy Kötter's equation, 17(10). In a second one it has been assumed that the normal stresses on both the wall and the surface of sliding are a function of the second power of the distance from the surface of the backfill, measured along the back of the lateral support and the surface of sliding respectively. The values of the constants contained in the functions have been determined in such a way that the conditions for the equilibrium of the sliding wedge are satisfied. In a third investigation another function has been selected, approximately expressing the distribution of the normal stresses over the boundaries of the sliding wedge. In spite of the differences between the fundamental assumptions, the values obtained by these methods for the ratio between the elevation of the centroid of the earth pressure and the height of the bank range between the narrow limits 0.48 and 0.56. They correspond to an angle of wall friction $\delta = 0$. However, the wall friction was found to have little influence on the location of the centroid of the pressure. Hence we are entitled to assume that the centroid is located approximately at midheight of the support and the corresponding pressure distribution is roughly parabolic, as shown on the right-hand side of Figure 17*c*. The investigation has also shown that an increase of the ratio H_a/H due to arching is associated with an increase of the horizontal pressure on the lateral support. A simple method of computing the intensity of the lateral pressure is described in Article 67. It is based on the assumption that the curve of sliding is a logarithmic spiral, which intersects the surface at right angles.

A general mathematical discussion of the influence of the wall movement on the earth pressure has been published by Jáky (1938).

CHAPTER VI

RETAINING WALL PROBLEMS

21. Definitions. Retaining walls are used to provide lateral support for masses of soil. The supported material is called the *backfill*. Figures 19 and 27 represent sections through the two principal types of retaining walls. The wall shown in Figure 19 is called a *gravity wall* because the wall depends on its own weight for stability against the horizontal thrust produced by the lateral earth pressure. On the other hand, the *cantilever retaining wall*, shown in Figure 27, derives part of its stability from the weight of the soil located above the footing at the back of the wall. The side of a retaining wall against which the fill is placed is called the *back of the wall*. The back may be plane or broken, and a plane back may be vertical or inclined (*battered*). The failure of a retaining wall can occur by tilting (*tilting failure*) or by sliding along its base parallel to its original position (*sliding failure*). Either type of failure of the wall is associated with the downward movement of a wedge-shaped body of soil (*abc* in Fig. 19) located immediately back of the wall. This body is called the *sliding wedge*.

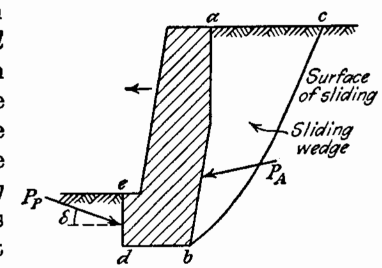

FIG. 19. Earth pressures acting on retaining wall at instant of failure.

22. Assumptions and conditions. Most of the theories of earth pressure are based on the following assumptions: The backfill of the wall is isotropic and homogeneous; the deformation of the backfill occurs exclusively parallel to a vertical plane at right angles to the back of the wall, and the neutral stresses in the backfill material are negligible. Any departure from these fundamental assumptions will be mentioned specifically. In this chapter it will be further assumed that the wall moves to a position which is located entirely beyond the boundary a_1b of the shaded area in Figure 14*c*. This is the deformation condition.

The width of the shaded area aa_1b in Figure 14*c* represents the amount by which the horizontal dimensions of the body of sand *abc* increase while the sand passes from its initial state of stress into that of plastic equilibrium. If a lateral support