Flow in Earth Dams

Need for Flow Information

- Estimate seepage through dam
- Estimate seepage through foundation soils
- Design of cutoffs and ground treatment programs
- Porewater pressure to determine up and downstream slope stability
- Prevention of piping failures

Piping Failures

$$FS = \frac{i_c}{i_m} \geq 3$$

$$i_c = \frac{\gamma'}{\gamma_{water}}$$

i_m = mobilized hydraulic gradient

Determine i_m from flow net

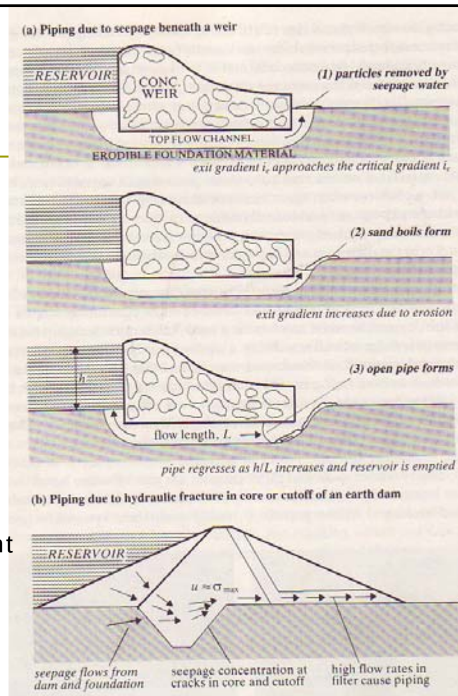


Figure 5.9 Piping failures.

Anisotropic K

$$\frac{\partial^2 h}{\partial x^2} k_x + \frac{\partial^2 h}{\partial z^2} k_z = 0$$

$$if \;\; k_x = k_z$$

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

Anisotropic $k_x > k_z$

$$\frac{\partial^2 h}{\partial x^2} k_x + \frac{\partial^2 h}{\partial z^2} k_z = 0$$

$$\frac{\partial^2 h}{\left(\frac{k_z}{k_x}\right)\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

$$\text{if } x_t = x\sqrt{\frac{k_z}{k_x}}$$

Anisotropic K

$$\frac{\partial^2 h}{\partial x_t^{\,2}} + \frac{\partial^2 h}{\partial z^2} = 0$$

$$v_x = -k' \frac{\partial h}{\partial x_t} = k \frac{\partial h}{\partial x}$$

$$k' = k_x \sqrt{\frac{k_z}{k_x}} = \sqrt{k_x k_z}$$

Anisotropic K

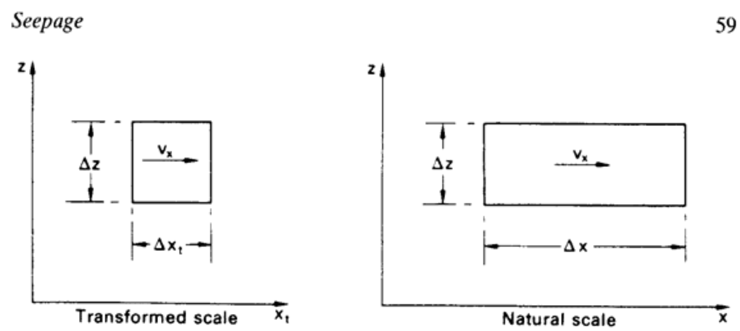


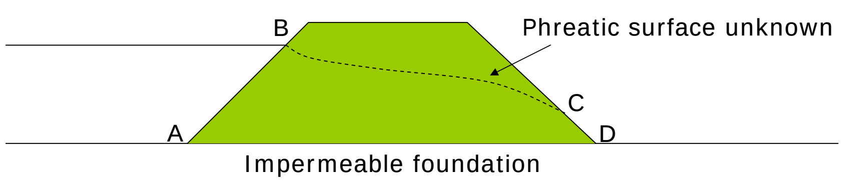
Fig. 2.12 Elemental flow net field.

Transform section with highest K

Flow Net Construction

Unconfined Flow System

- Phreatic line unknown
  - (air-water interface)



AB –equipotential line AD – flow line

BC is not equipotential or flow line

Unconfined flow

Two basic methods to determine phreatic surface:

1) Draw trial flow net
- See Cedergren Seepage Drainage and Flownets – posted on web.
2) Numerical solution based on parabola

Numerical Solution

Dupuit Solution

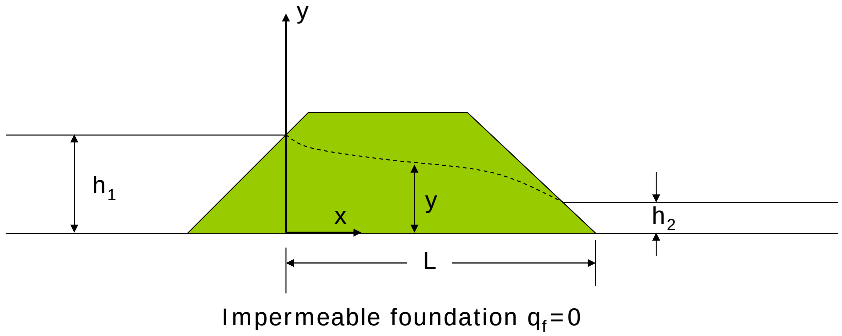
Modified Laplace equation

Assumes:

a) Flow lines to be nearly horizontal
b) Hydraulic gradient of the flow is equal to the slope of the phreatic surface

$$i = \frac{\partial y}{\partial x}$$

Dupuit Solution



Dupuit Solution

- Top phreatic surface defined by

$$y = \sqrt{h_1^{\ 2} - \left(h_1^{\ 2} - h_2^{\ 2}\right)\frac{x}{L}}$$

$$q = -k\left(\frac{h_1^{\ 2} - h_2^{\ 2}}{2L}\right)$$

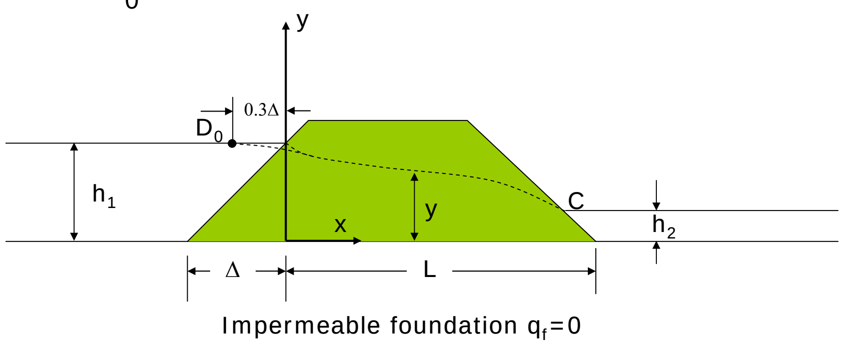
Dupuit Solution

- Does not take into account:
  - Slope geometry
  - Entrance or exit conditions

  (seepage surface is missing)

Casagrande Entrance Condition

- Move parabola by 0.3Δ and draw parabola from D_0 to C



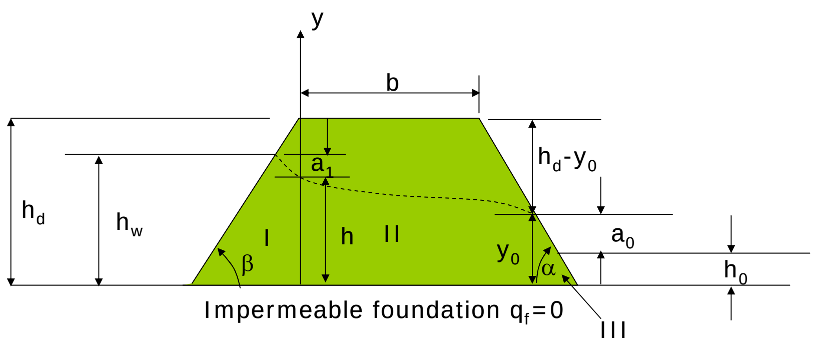
Exit Condition

Different Methods Available:

Graphical solutions:

1. Schaffemak and Van Iterson
2. Casagrande
3. Pavovsky's solution

Pavovsky's Solution



$$q_I = q_{II} = q_{III}$$

Pavovsky's Solution

$$\frac{a_0}{m_1} = \left(\frac{h_w - h}{m}\right) \ln \frac{h_d}{h_d - h}$$

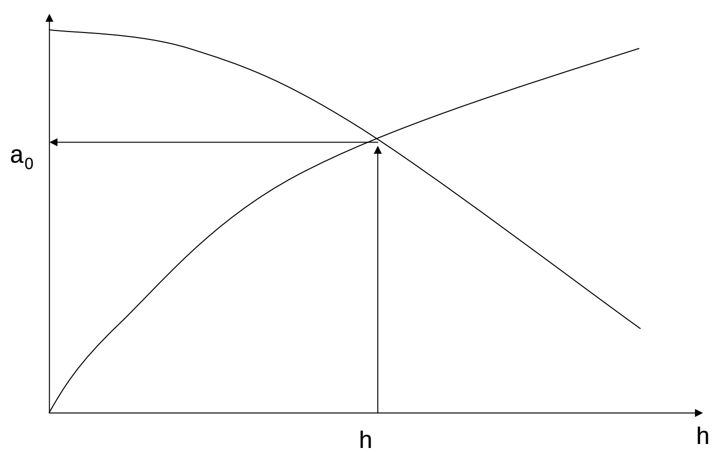
where m_1 = cotanα, m = cotanβ

$$a_0 = \frac{b}{m_1} + h_d - \sqrt{\left(\frac{b}{m_1} + h_d{}^2 - h^2\right)}$$

Use trial and error to solve non-liner equations

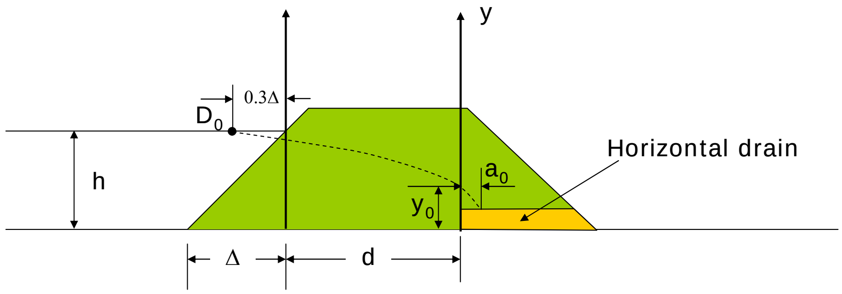
Assume different values of h and calculate a_0

Pavovsky's Solution



Dam with Horizontal Drain

- Dupuit Solution



Dam with Horizontal Drain

$$y_0 = \sqrt{d^2 + h^2} - d$$

$$a_0 = \frac{y_0}{2}$$

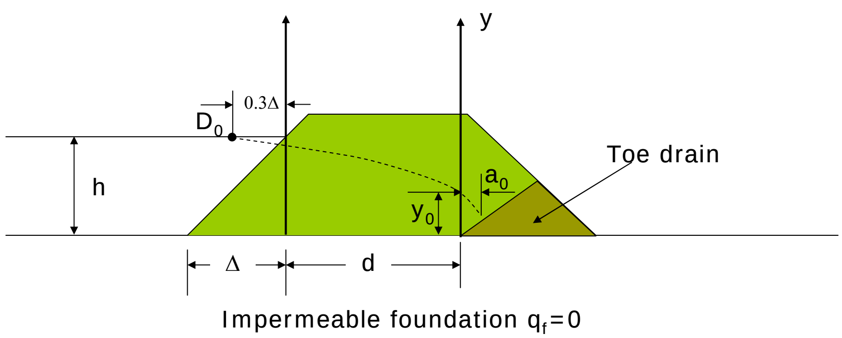
$$q = ky_0 = k\sqrt{d^2 + h^2} - d$$

Horizontal Drains

Good Practice Rule

Length of horizontal drain from the down stream slope should be at least H/3 where H is the height of the dam

Dams with Triangular Toe Drain

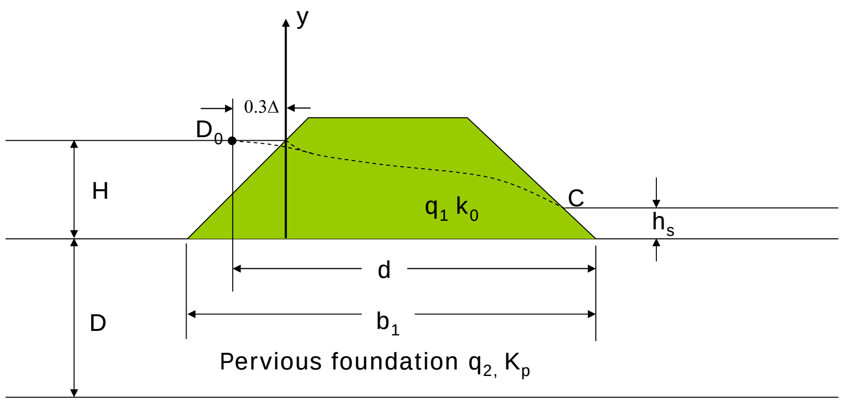


Assume y_0 and a_0. Error on safe side so that design phreatic surface will be higher than expected…this will increase porewater pressure and lower factor of safety.

Casagrande proposed corrections for toe drains – this can be ignored

Dams on Pervious Foundation

See Canal and River Levees by Pavol Peters Elsevier publishing 1982



Dams on Pervious Foundation

$$q = q_1 + q_2$$

$$q = \frac{k_0\left(H^2 - h_s^{\,2}\right)}{2d} + k_p \frac{HD}{b_1 n}$$

B_1/D	20	5	4	3	2	1
n	1.15	1.18	1.23	1.30	1.44	1.87

Dams on Pervious Foundation

Analysis assumes

$$k = k_0 = k_p$$

K in dam ~k in foundation