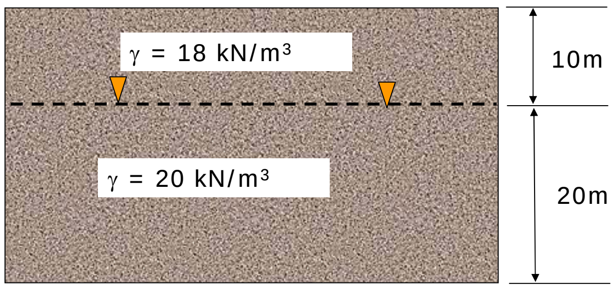
Vertical Stress in a Soil Mass

Forces that Increase Vertical Stress in Soil Mass

- Weight of soil (effective stress)
- Surface loads
  - Fill large area
  - Point loads:
    - Hydro pole, light stand, column, etc
  - Lines loads
    - Rack or rail loading, strip foundation
  - Rectangular area
    - Raft or rectangular footing
  - Circular area
    - tank
  - Earth embankment
    - Road, railway, fill, ice, etc.

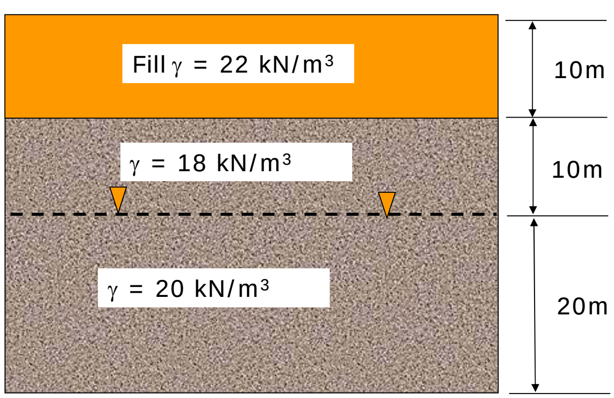
Fill or Surcharge over Large Area

CASE I No flow and no surcharge



Fill or surcharge over large area

CASE II 10m of fill added



Fill or Surcharge over large area

$$\sigma_{fill} = \gamma_{fill} H = \Delta\sigma$$

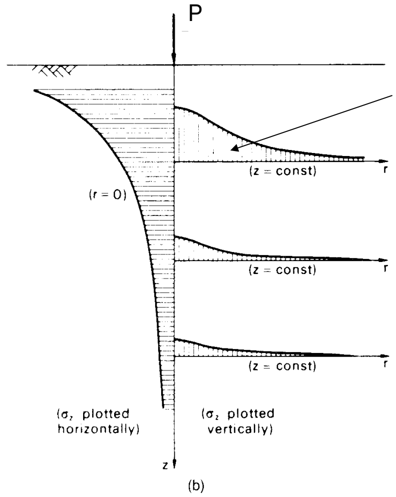
Note: $\Delta\sigma_{fill}$ = constant with depth

Glaciers over N.A during ICE AGE increased vertical stress in soil and rock. This stress is now gone.

Boussinesq 1885 Point Load Solution

- Uses elastic theory to get change in stress with depth below point load
- Solution based on:
  - Linear elastic, homogeneous, isotropic medium with semi-infinite depth

Point Load



Integration of area at a given depth must equal the applied surface area

With depth peak gets smaller however stress spreads over larger area

Fig. 5.5 (a) Stresses due to point load. (b) Variation of vertical stress due to point load.

Point Load

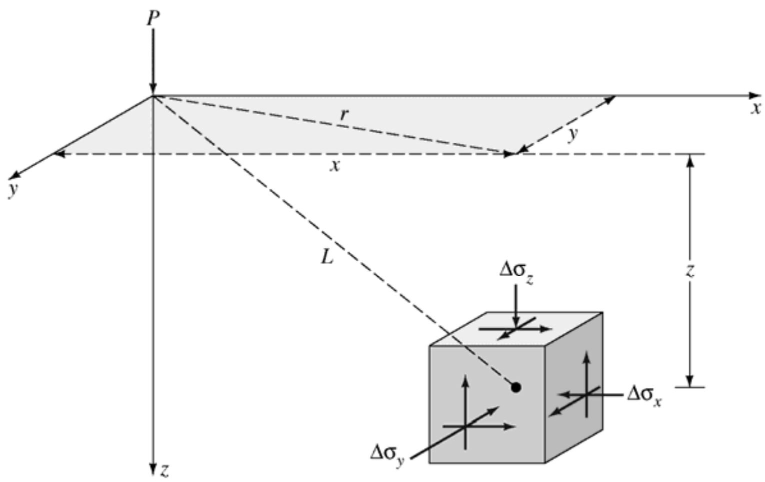


Figure 6.11 Stresses in an elastic medium caused by a point load

Point Load

$$\Delta\sigma_x = \frac{P}{2\pi}\left\{\frac{3x^2z}{L^5} - (1-2\mu)\left[\frac{x^2-y^2}{Lr^2(L+z)} + \frac{y^2z}{L^3r^2}\right]\right\} \quad (6.15)$$

$$\Delta\sigma_y = \frac{P}{2\pi}\left\{\frac{3y^2z}{L^5} - (1-2\mu)\left[\frac{y^2-x^2}{Lr^2(L+z)} + \frac{x^2z}{L^3r^2}\right]\right\} \quad (6.16)$$

$$\Delta\sigma_z = \frac{3P}{2\pi}\frac{z^3}{L^5} = \frac{3P}{2\pi}\frac{z^3}{(r^2+z^2)^{5/2}} \quad (6.17)$$

where $r = \sqrt{x^2 + y^2}$

$L = \sqrt{x^2 + y^2 + z^2} = \sqrt{r^2 + z^2}$

μ = Poisson's ratio

Point Load

Note that Eqs. (6.15) and (6.16), which are the expressions for horizontal normal stresses, are dependent on Poisson's ratio of the medium. However, the relationship for the vertical normal stress, $\Delta\sigma_z$, as given by Eq. (6.17), is independent of Poisson's ratio. The relationship for $\Delta\sigma_z$ can be rewritten in the following form:

$$\Delta\sigma_z = \frac{P}{z^2}\left\{\frac{3}{2\pi}\frac{1}{[(r/z)^2+1]^{5/2}}\right\} = \frac{P}{z^2}I_1 \quad (6.18)$$

where $I_1 = \dfrac{3}{2\pi}\dfrac{1}{[(r/z)^2+1]^{5/2}}$. (6.19)

The variation of I_1 for various values of r/z is given in Table 6.1.

Typical values of Poisson's ratio for various soils are listed in Table 6.2.

Table 6.1 Variation of I_1 [Eq. (6.18)]

r/z	I_1	r/z	I_1
0	0.4775	0.9	0.1083
0.1	0.4657	1.0	0.0844
0.2	0.4329	1.5	0.0251
0.3	0.3849	1.75	0.0144
0.4	0.3295	2.0	0.0085
0.5	0.2733	2.5	0.0034
0.6	0.2214	3.0	0.0015
0.7	0.1762	4.0	0.0004
0.8	0.1386	5.0	0.00014

Table 6.2 Representative values of Poisson's ratio

Type of soil	Poisson's ratio, μ
Loose sand	0.2–0.4
Medium sand	0.25–0.4
Dense sand	0.3–0.45
Silty sand	0.2–0.4
Soft clay	0.15–0.25
Medium clay	0.2–0.5

Line Load

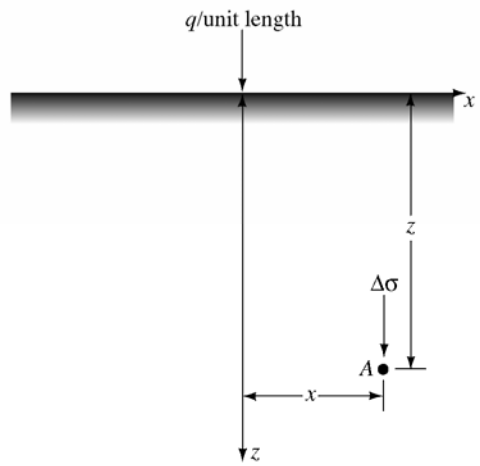


Figure 6.12
Line load over the surface of a semiinfinite soil mass

Line Load

$$\Delta\sigma = \frac{2qz^3}{\pi(x^2 + z^2)^2} \qquad (6.20)$$

The preceding equation can be rewritten as

$$\Delta\sigma = \frac{2q}{\pi z[(x/z)^2 + 1]^2}$$

or

$$\frac{\Delta\sigma}{(q/z)} = \frac{2}{\pi\left[\left(\frac{x}{z}\right)^2 + 1\right]^2} \qquad (6.21)$$

Table 6.3 Variation of $\Delta\sigma/(q/z)$ with x/z [Eq. (6.21)]

x/z	$\frac{\Delta\sigma}{q/z}$	x/z	$\frac{\Delta\sigma}{q/z}$
0	0.637	0.7	0.287
0.1	0.624	0.8	0.237
0.2	0.589	0.9	0.194
0.3	0.536	1.0	0.159
0.4	0.473	1.5	0.060
0.5	0.407	2.0	0.025
0.6	0.344	3.0	0.006

Strip Load

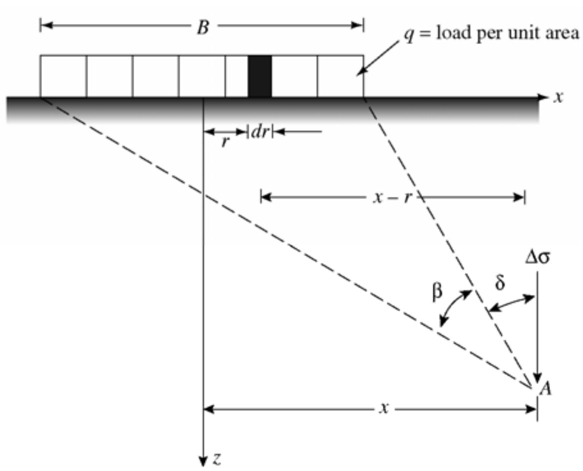


Figure 6.13 Vertical stress caused by a flexible strip load (*Note:* Angles measured in counter-clockwise direction are taken as positive.)

Strip Load

$-B/2$ to $+B/2$, or

$$\Delta\sigma = \int d\sigma = \int_{-B/2}^{+B/2}\left(\frac{2q}{\pi}\right)\left\{\frac{z^3}{[(x-r)^2+z^2]^2}\right\}dr$$
$$= \frac{q}{\pi}\left\{\tan^{-1}\left[\frac{z}{x-(B/2)}\right] - \tan^{-1}\left[\frac{z}{x+(B/2)}\right] - \frac{Bz[x^2-z^2-(B^2/4)]}{[x^2+z^2-(B^2/4)]^2+B^2z^2}\right\} \quad (6.2$$

Equation (6.23) can be simplified to the form

$$\Delta\sigma = \frac{q}{\pi}[\beta + \sin\beta\cos(\beta+2\delta)] \quad (6.2$$

The angles β and δ are defined in Figure 6.13.

Table 6.4 shows the variation of $\Delta\sigma/q$ with $2z/B$ for $2x/B$ equal to 0, 0.5, 1.0, 1.5 and 2.0. This table can be conveniently used to calculate the vertical stress at a poi caused by a flexible strip load.

Table 6.4 Variation of $\Delta\sigma/q$ with $2z/B$ and $2x/B$

	2x/B				
2z/B	**0**	**0.5**	**1.0**	**1.5**	**2.0**
0	1.000	1.000	0.500	—	—
0.5	0.959	0.903	0.497	0.089	0.019
1.0	0.818	0.735	0.480	0.249	0.078
1.5	0.668	0.607	0.448	0.270	0.146
2.0	0.550	0.510	0.409	0.288	0.185
2.5	0.462	0.437	0.370	0.285	0.205
3.0	0.396	0.379	0.334	0.273	0.211
3.5	0.345	0.334	0.302	0.258	0.216
4.0	0.306	0.298	0.275	0.242	0.205
4.5	0.274	0.268	0.251	0.226	0.197
5.0	0.248	0.244	0.231	0.212	0.188

Below centre of uniformly loaded circular area

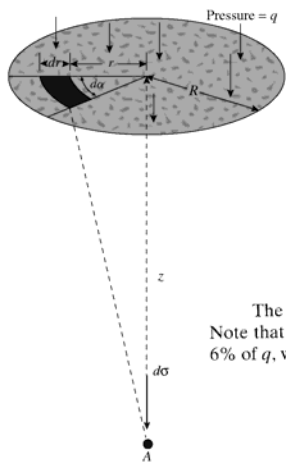


Figure 6.15
Vertical stress below the center of a uniformly loaded flexible circular area

$$\Delta\sigma = q\left\{1 - \frac{1}{[(R/z)^2 + 1]^{3/2}}\right\} \qquad (6.26)$$

The variation of $\Delta\sigma/q$ with z/R obtained from Eq. (6.26) is given in Table 6.5. Note that the value of $\Delta\sigma$ decreases rapidly with depth, and, at $z = 5R$, it is about 6% of q, which is the intensity of pressure at the ground surface.

Table 6.5 Variation of $\Delta\sigma/q$ with z/R [Eq. (6.26)]

z/R	$\Delta\sigma/q$	z/R	$\Delta\sigma/q$
0	1	1.0	0.6465
0.02	0.9999	1.5	0.4240
0.05	0.9998	2.0	0.2845
0.10	0.9990	2.5	0.1996
0.2	0.9925	3.0	0.1436
0.4	0.9488	4.0	0.0869
0.5	0.9106	5.0	0.0571
0.8	0.7562		

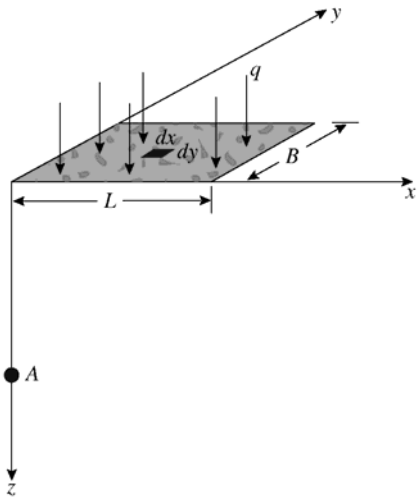


Figure 6.16
Vertical stress below the corner of a uniformly loaded flexible rectangular area

Rectangular Loaded Area

$$\Delta\sigma = \int d\sigma = \int_{y=0}^{B}\int_{x=0}^{L} \frac{3qz^3(dx\,dy)}{2\pi(x^2+y^2+z^2)^{5/2}} = qI_2 \qquad (6.2$$

$$\text{where } I_2 = \frac{1}{4\pi}\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+m^2n^2+1}\left(\frac{m^2+n^2+2}{m^2+n^2+1}\right) + \tan^{-1}\left(\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2-m^2n^2+1}\right)\right] \qquad (6.3$$

$$m = \frac{B}{z} \qquad (6.3$$

$$n = \frac{L}{z} \qquad (6.3$$

B is always shortest dimension

Fadum Chart

$$\Delta\sigma = q\, I_2$$

$$m = \frac{B}{z}$$

$$n = \frac{L}{z}$$

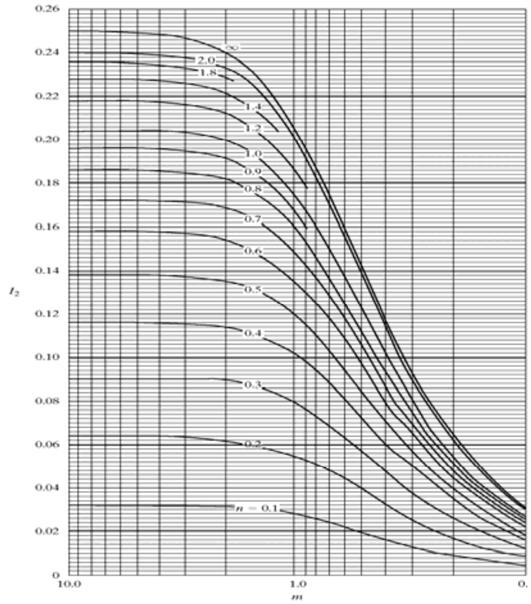


Figure 6.17 Variation of I_2 with m and n

$$\Delta\sigma_c = qI_c \qquad (6.3$$

where

$$I_c = f(m_1, n_1) \qquad (6.3$$

$$m_1 = \frac{L}{B} \qquad (6.3$$

and

$$n_1 = \frac{z}{\frac{B}{2}} \qquad (6.3$$

Table 6.6 gives the variation of I_c with m_1 and n_1.

Table 6.6 Variation of I_c with m_1 and n_1 [Eq. (6.34)]

	m_1									
n_1	1	2	3	4	5	6	7	8	9	10
0.20	0.994	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997
0.40	0.960	0.976	0.977	0.977	0.977	0.977	0.977	0.977	0.977	0.977
0.60	0.892	0.932	0.936	0.936	0.937	0.937	0.937	0.937	0.937	0.937
0.80	0.800	0.870	0.878	0.880	0.881	0.881	0.881	0.881	0.881	0.881
1.00	0.701	0.800	0.814	0.817	0.818	0.818	0.818	0.818	0.818	0.818
1.20	0.606	0.727	0.748	0.753	0.754	0.755	0.755	0.755	0.755	0.755
1.40	0.522	0.658	0.685	0.692	0.694	0.695	0.695	0.696	0.696	0.696
1.60	0.449	0.593	0.627	0.636	0.639	0.640	0.641	0.641	0.641	0.642
1.80	0.388	0.534	0.573	0.585	0.590	0.591	0.592	0.592	0.593	0.593
2.00	0.336	0.481	0.525	0.540	0.545	0.547	0.548	0.549	0.549	0.549
3.00	0.179	0.293	0.348	0.373	0.384	0.389	0.392	0.393	0.394	0.395
4.00	0.108	0.190	0.241	0.269	0.285	0.293	0.298	0.301	0.302	0.303
5.00	0.072	0.131	0.174	0.202	0.219	0.229	0.236	0.240	0.242	0.244
6.00	0.051	0.095	0.130	0.155	0.172	0.184	0.192	0.197	0.200	0.202
7.00	0.038	0.072	0.100	0.122	0.139	0.150	0.158	0.164	0.168	0.171
8.00	0.029	0.056	0.079	0.098	0.113	0.125	0.133	0.139	0.144	0.147
9.00	0.023	0.045	0.064	0.081	0.094	0.105	0.113	0.119	0.124	0.128
10.00	0.019	0.037	0.053	0.067	0.079	0.089	0.097	0.103	0.108	0.112

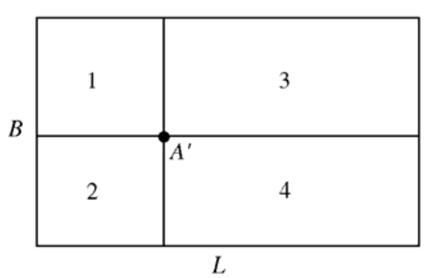


Figure 6.18
Increase of stress at any point below a rectangularly loaded flexible area

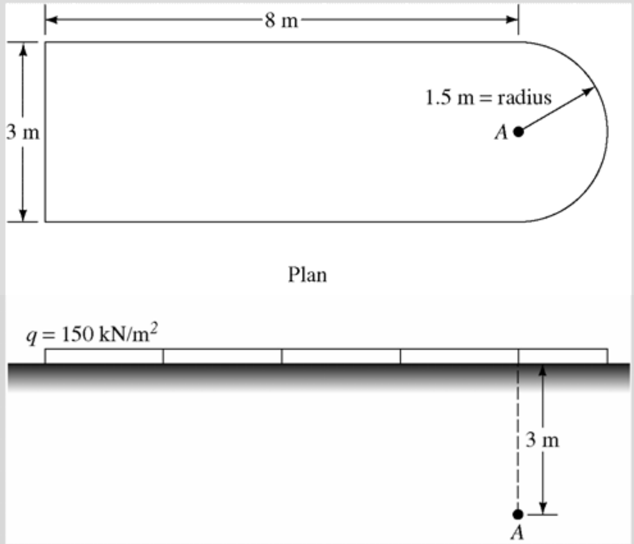


Figure 6.19

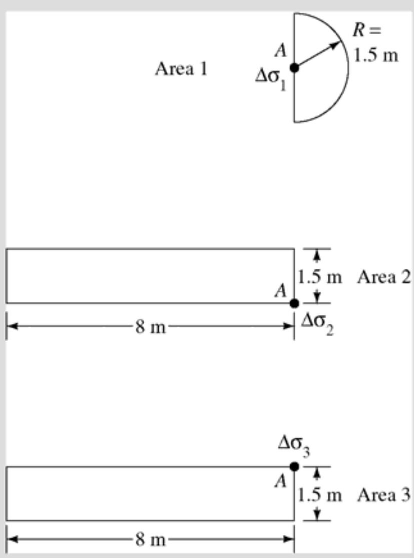


Figure 6.20

Newmark's Influence Chart

Influence Chart for Vertical Stress

Newmark [5.12] constructed an influence chart, based on the Boussinesq solution, enabling the vertical stress to be determined at any point below an area of any shape carrying a uniform pressure q. The chart (Fig. 5.11) consists of influence areas, the boundaries of which are two radial lines and two circular arcs. The loaded area is drawn on tracing paper to a scale such that the length of the scale line on the chart represents the depth z at which the vertical stress is required. The position of the loaded area on the chart is such that the point at which the vertical stress is required is at the centre of the chart. For the chart shown in Fig. 5.11 the influence value is 0·005, i.e. each influence area represents a vertical stress of 0·005q. Hence, if the number of influence areas covered by the scale drawing of the loaded area is N, the required vertical stress is given by

$$\sigma_z = 0{\cdot}005\,Nq$$

Newmark's Chart

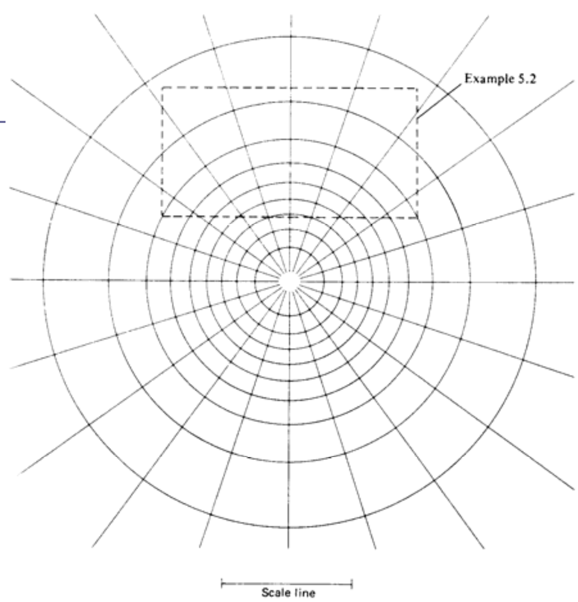


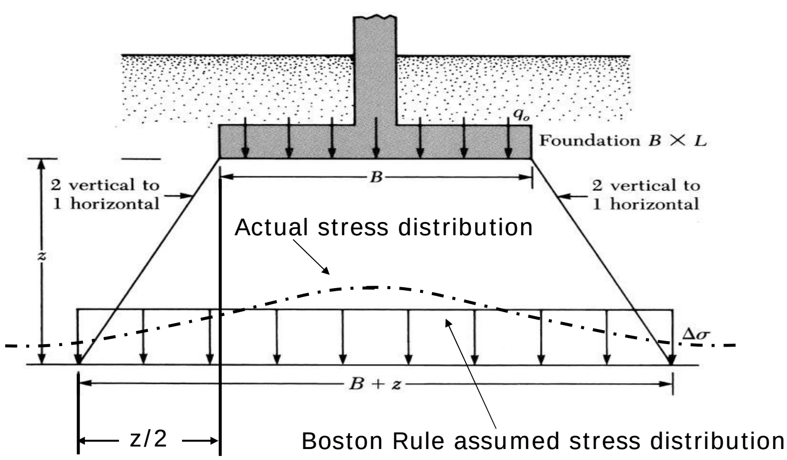
Fig. 5.11 Newmark's influence chart for vertical stress. Influence value per unit pressure = 0.005. (Reproduced from N.M. Newmark (1942) *Influence Charts for Computation of Stresses in Elastic Foundations*, University of Illinois Bulletin No. 338, by permission of Professor Newmark.)

Boston Rule (Approximate method)

- For uniform footing (B x L) we can estimate the change in vertical stress with depth using the Boston Rule

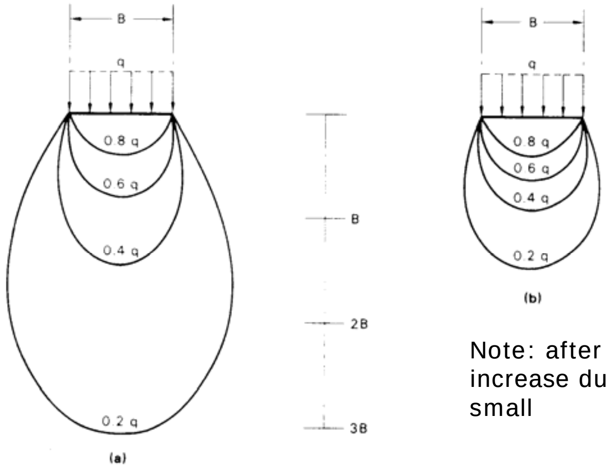
- Assumes stress at depth is constant below foundation influence area

Boston Rule (Approximate Method)



$\Delta\sigma_z = q_o(BxL) / (B+z)(L+z) = P/(B+z)(L+z)$

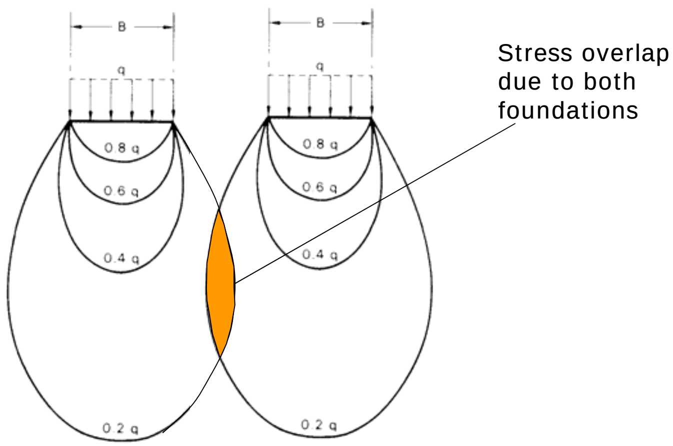
Stress Influence Area



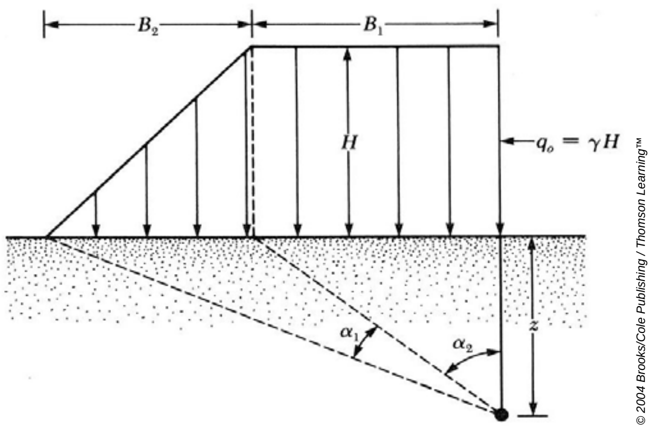
Note: after 2B stress increase due to q is very small

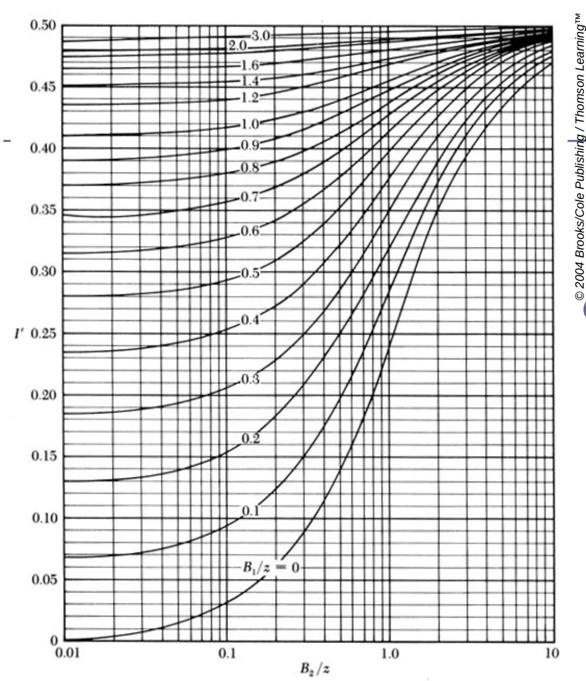
Fig. 5.8 Contours of equal vertical stress: (a) under strip area, (b) under square area.

Foundation Stress Influence



Embankment Loading





Influence value *I'* for embankment loading

(after Osterberg, 1957)

$$\Delta\sigma_z = 2I'q_0$$