5. Determination of the discharge capacities of porous aggregate drains, chimney drains, and sand-filled wells. (These determinations also make use of the *discharge velocity,* defined in Sec. 3.4). Methods of correcting for turbulent flow are given in Sec. 3.7.

The relationships represented by Darcy's law, though simple, represent some of the most powerful tools available to the soils engineer and the drainage engineer. Unfortunately their great benefits are not always realized. Practical examples of the application of Darcy's law in the solution of countless everyday engineering problems relating to seepage and drainage are to be found throughout this book. For instance, simple calculations given in this new edition help explain why serious groundwater contamination problems went on so long without being noticed (see Sec. 11.1).

3.3 FLOW NETS

Basic Solutions to Seepage Problems

The flow of water through soil is but one of a number of forms of streamline flow that obey similar fundamental relationships and can be represented by the Laplace equation. Muskat (1937) demonstrates that the hydrodynamics of steady-state fluid flow through porous media follows the same basic laws as the problems of steady-state heat flow, electrostatics, and current flow in continuous conductors. Other types of problem, such as certain cases in the theory of torsion of elastic rods and the flow of viscous liquids, are also governed by Laplace's equation.

To develop the Laplace equations for flow of water through porous media, let the following be assumed:

1. The soil is homogeneous.
2. The voids are completely filled with water.
3. No consolidation or expansion of the soil takes place.
4. The soil and water are incompressible.
5. Flow is laminar and Darcy's law is valid.

It can then be shown (Terzaghi, 1943) that the quantity of water entering an element must equal that leaving and the equation of continuity takes the form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (3.6)$$

The terms u, v, and w are discharge velocity components in directions x, y, and z in cartesian coordinates.

According to Darcy's law ($v_d = ki$), the components of the discharge velocity are

$$u = -k\frac{\partial h}{\partial x}, \qquad v = -k\frac{\partial h}{\partial y}, \qquad w = -k\frac{\partial h}{\partial z}$$

Substituting in Eq. 3.6,

$$\partial\frac{-k(\partial h/\partial x)}{\partial x} + \partial\frac{-k(\partial h/\partial y)}{\partial y} + \partial\frac{-k(\partial h/\partial z)}{\partial z} = 0 \tag{3.7}$$

and if k is a constant

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0 \tag{3.8}$$

This is the common form of the Laplace equation for three-dimensional flow of water through porous media. In two dimensions it has the form

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0 \tag{3.9}$$

Equation 3.9 may be represented by two families of curves that intersect at right angles to form a pattern of "square" figures known as a *flow net.* One set of lines is called the *streamlines* or *flow lines,* the other the *equipotentials.* The flow lines represent paths along which water can flow through a cross section. The equipotential lines are lines of equal energy level or head.

Although mathematical solutions have been developed for a number of cases of flow, the solutions are cumbersome and often approximate. Leliavsky (1955) says, ". . . the analytical method, although rigorously precise, is not universally applicable, because the number of known functions on which it depends is limited. Moreover, except in a few elementary cases, the analytical method lies beyond the mathematics of practising design offices."

For those who wish to pursue the mathematical approach Harr (1962) presents an interesting treatment of seepage theory, giving both rigorous and approximate mathematical solutions to seepage under weirs and other structures. He describes conformal mapping, the velocity hodograph, and other special mapping techniques, such as the Zhukovsky functions and the Schwarz-Christoffel transformations.

Solutions to flow problems are also obtained by the use of the *electric analogy,* small-scale *viscous fluid* models, (see Fig. 3.27), sand models, and trial-and-error sketching methods. Sand models, with colored dye inserted, give a graphic picture of seepage that is convincing to the skeptic (see Fig. 3.2). The electric analogy is adaptable to two- and three-dimensional problems.

Mathematical solutions to practical multi-permeability cross sections can be extremely complex and difficult to interpret and are little more than fancy exercises unless they can be put to practical use. Computerized solutions to the Laplace equation are being developed, and some very good ones are available to those having access to the programs. Biedenham and Tracy (1987) have prepared a technical report, No. ITL-87-6, a Users Guide Finite Element Method Seepage Package for solving certain steady-state problems. Inquiries should be made to the Waterways Experiment Station, U.S. Army Engineer Division, Vicksburg, Miss., 39180-0631. Using the program requires knowledge of computerized solution methods and thorough familiarity with the details for setting up individual problems. Other programs have been developed. At the present time (1988) there is no clear-cut source of information on all programs available and under what conditions, so anyone wishing to make use of programs must do some searching on his or her own to try to determine which are available and under what terms.

In a thorough review of numerical methods and computer solutions in ground engineering, Christian (1987) points out that there are a number of computer programs available to the general user for solving problems of fluid flow. As long as non-linear properties or phreatic surfaces are not involved, general-purpose finite element programs can be used, he says. Many of these programs are capable of treating problems of thermal conductivity, which is mathematically identical to the problem of flow through porous media when expressed in terms of total head. He also points out that a large number of programs have been written specifically for flow through porous media and many are available from geological surveys or other groups working with groundwater hydrology. Under "Conclusions" he says, "The engineer contemplating employing numerical methods should be familiar with the actual physical problem to be investigated, the numerical techniques to be used, and the limitations and capabilities of the software." He emphasizes that if a person contemplating numerical solutions is not familiar with all of these aspects, "he should consult someone who is."

Christian also says that numerical and computer methods have become so complicated that they "begin to form a discipline within the larger engineering field." While problems exist in the preparation of input to and interpreting output from computer programs, and shortcomings in reliable, documented software exist, ground engineers can expect important developments (in these methods) over the next few years, according to Christian.

Weber and Hassan (1972) discuss methods of developing models to simulate the behavior of groundwater basins under changed conditions brought on by increased pumping or other management changes. One approach divides an entire groundwater basin into a mesh of subareas in which transmission and storage factors can be estimated. Based on a knowledge of past behavior patterns, these models predict changes under the different conditions that may develop in the future.

When a problem is set up for electronic computer or model analysis, exactly repeatable answers are obtained. It must be remembered, however, that the answers are no better than the input. Useful though these methods may be the graphic sketching methods for solving the Laplace equation are preferred by many, because they are so extremely versatile. Essentially all of the flow nets presented in this book were developed by sketching methods. Step-by-step procedures for sketching flow nets are given in Chapter 4.

Any flow net must meet certain requirements, of which the following are basic:

1. Flow lines and equipotential lines must intersect at right angles to form areas that are basically squares.
2. Certain entrance and exit requirements must be met.
3. A basic deflection rule must be followed in passing from a soil of one permeability to a soil of a different permeability.
4. Adjacent equipotentials have equal head losses.
5. The same quantity of seepage flows between adjacent pairs of flow lines.

The last two are fundamental requirements *indirectly* entering into the construction of flow nets. Another requirement may be stated: the quantity of seepage flowing through a section must be constant throughout the section unless additional water enters or some is removed by drains. It will be seen that in dams and other cross sections in which seepage emerges on a slope a portion of the flow net is cut off and this rule does not apply.

At first glance the sketching method may appear to present obstacles to a beginner, but once the initial obstacles are overcome one seldom resorts to other means of solving seepage problems. The frequent, thoughtful examination of well-constructed flow nets is helpful in becoming familiar with their general shapes. Numerous examples may be found in this text.

Flow Lines and Equipotential Lines

When water is forced through porous soil, particles of water flow through all of its interconnected pore spaces. Although the number of individual paths is nearly infinite, the flow lines drawn in any flow net are but a few of the many paths that water can take in flowing through a cross section. The nature of flow lines is illustrated in simple form in Figure 3.1. A small volume of sand is contained within a horizontal tube, to which reservoirs are attached at both ends. If a difference in head h of any magnitude exists, water will flow through the soil from left to right. If several small diameter tubes are inserted a small distance into the sand at the left reservoir and dye is fed into these tubes, colored streaks, marking the *flow lines,* will form. An infinite number of lines is possible. Obviously, if dye is inserted at a great many locations, the whole section will become colored and the individual paths will be lost.

In Figure 3.1 the flow lines are straight because the confining boundaries

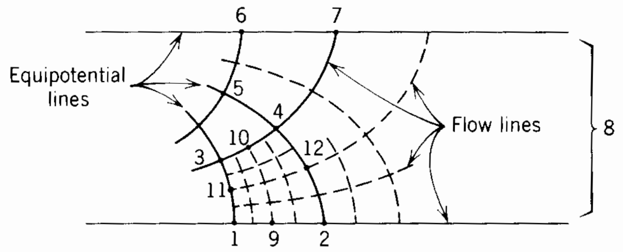

FIG. 3.5 Subdivision of peculiar "squares" provides check on accuracy of original lines. (After A. Casagrande, *Seepage Through Dams,* 1937.)

Boundary Conditions

Casagrande (1937) describes important conditions that must be met by flow nets at points of entry, discharge, and transfer across boundaries between dissimilar soils. If flow is beneath a sheet pile wall or through the foundation of a dam or other structure impounding water, all boundary conditions are fixed, but if the seepage is through an earth dam, levee, or other embankment the *upper boundary* or upper *line of seepage* is not known in advance of the flow net construction. At the uppermost line of seepage equipotential lines must intersect the free water surface at equal vertical intervals (Fig. 3.6). This requirement permits determination of the free surface or uppermost line of seepage *simultaneously while a flow net is being constructed.* The step-by-step procedure is shown in Chapter 4.

Important entrance, discharge, and boundary conditions that must be met by flow nets are given in Figure 3.7. An important condition existing at the boundaries between soils of different permeabilities is discussed in detail in the following section.

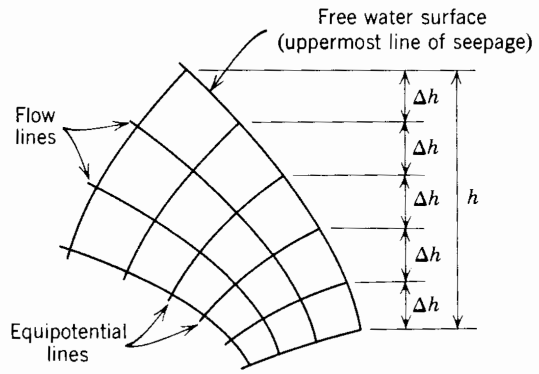

FIG. 3.6 General condition for line of seepage. (After A. Casagrande, *Seepage Through Dams,* 1937.)

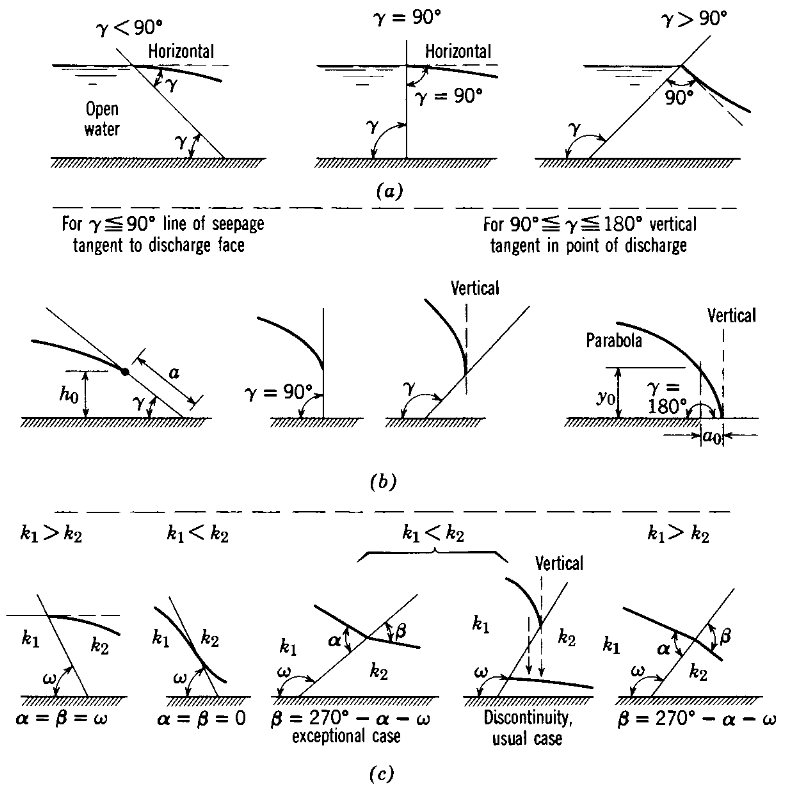

FIG. 3.7 Entrance, discharge, and transfer conditions of line of seepage. (*a*) Conditions for point of entrance of line of seepage. (*b*) Conditions for point of discharge of line of seepage. (*c*) Deflection of line of seepage at boundary between soils of different permeability. (After A. Casagrande, *Seepage Through Dams,* 1937.)

Flow through Sections of More Than One Permeability

Many practical seepage and drainage problems can be studied by constructing flow nets for sections with a single permeability. In some instances, however, a great deal can be learned by studying seepage patterns in cross sections with soils of more than one permeability. Surprisingly, such studies have revealed important shortcomings in some commonly accepted beliefs about seepage and drainage. *The study of composite cross sections is one of the most worthwhile and rewarding applications of the flow net.* At first glance flow nets for sections with more than one permeability may appear rather formidable, but attention to the transfer conditions at boundaries between soils of different permeabilities (Eqs. 3.10 and 3.11) greatly facilitates their construction.

When water flows across a boundary between dissimilar soils, the flow lines bend much in the way that light rays are deflected in passing from air into