effective stress on a soil was exceeded in the past (an overconsolidated soil), it would behave elastically (approximately) for stresses less than its past maximum effective stress.

Practical Examples

Interactive Problem Solving
Access Chapter 6, click on Problem Solver on the sidebar and then solve one-dimensional consolidation problems interactively.

EXAMPLE 6.13 ***Lateral Stress During Soil Consolidation in the Lab***

A soil was consolidated in an oedometer to a vertical stress of 100 kPa and then unloaded incrementally to 50 kPa. The excess porewater pressure is zero. If the frictional soil constant ϕ'_{cs} is 25°, determine the lateral stress.

Strategy The soil in this case becomes overconsolidated—the past maximum effective stress is 100 kPa and the current effective stress is 50 kPa. You need to find K_o^{nc} and then K_o^{oc} using the OCR of your soil. (See Section 5.10.)

Step 1: Calculate K_o^{nc}.

Eq. (5.51): $K_o^{nc} = 1 - \sin\phi'_{cs} = 1 - \sin 25 = 0.58$

Step 2: Calculate OCR.

$$\text{OCR} = \frac{\sigma'_{zc}}{\sigma'_{zo}} = \frac{100}{50} = 2$$

Step 3: Calculate K_o^{oc}.

Eq. (5.52): $K_o^{oc} = K_o^{nc}(\text{OCR})^{1/2} = 0.58(2)^{1/2} = 0.82$

Step 4: Calculate the lateral effective stress.

$$\sigma'_x = K_o^{oc}\sigma'_z = 0.82 \times 50 = 41\,\text{kPa}$$

Step 5: Calculate the lateral total stress.

$$\sigma_x = \sigma'_x + \Delta u = 41 + 0 = 41\,\text{kPa}$$

■

EXAMPLE 6.14 ***Consolidation Settlement Due to a Foundation***

A foundation for an oil tank is proposed for a site with a soil profile as shown in Fig. E6.14a. A specimen of the fine-grained soil, 75 mm in diameter and 20 mm thick, was tested in an oedometer in a laboratory. The initial water content was 62% and $G_s = 2.7$. The vertical stresses were applied incrementally—each increment remaining on the specimen until the porewater pressure change was negligible. The cumulative settlement values at the end of each loading step are as follows:

Vertical stress (kPa)	15	30	60	120	240	480
Settlement (mm)	0.10	0.11	0.21	1.13	2.17	3.15

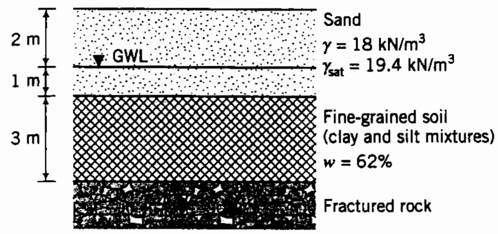

FIGURE E6.14a

The time–settlement data when the vertical stress was 200 kPa are:

Time (min)	0	0.25	1	4	9	16	36	64	100
Settlement (mm)	0	0.22	0.42	0.6	0.71	0.79	0.86	0.91	0.93

The tank, when full, will impose vertical stresses of 90 kPa and 75 kPa at the top and bottom of the fine-grained soil layer, respectively. You may assume that the vertical stress is linearly distributed in this layer.

(a) Determine the primary consolidation settlement of the fine-grained soil layer when the tank is full.

(b) Calculate and plot the settlement–time curve.

Strategy To calculate the primary consolidation settlement you need to know C_c and C_r or m_v, and σ'_{zo}, $\Delta\sigma$, and σ'_{zc}. Use the data given to find the values of these parameters. To find time for a given degree of consolidation, you need to find C_v from the data.

Solution 6.14

Step 1: Find C_v using the root time method.
Use the data from the 240 kPa load step to plot a settlement versus $\sqrt{\text{time}}$ curve as depicted in Fig. E6.14b. Follow the procedures set out in Section 6.7 to find C_v. From the curve, $t_{90} = 1.2$ min.

Height of sample at beginning of loading = 20 − 1.2 = 18.8 mm

Height of sample at end of loading = 20 − 2.17 = 17.83 mm

$$\text{Equation (6.1)}: \quad H_{dr} = \frac{H_o + H_f}{4} = \frac{18.8 + 17.83}{4} = 9.16\,\text{mm}$$

$$\text{Equation (6.31)}: \quad C_v = \frac{T_v H_{dr}^2}{t_{90}} = \frac{0.848 \times (9.16)^2}{1.2} = 59.3\,\text{mm}^2/\text{min}$$

$$= 59.3 \times 10^{-6}\,\text{m}^2/\text{min}$$

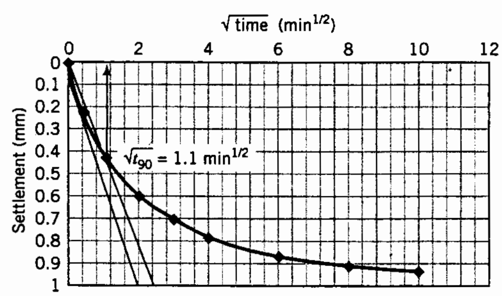

FIGURE E6.14b

Step 2: Determine the void ratio at the end of each load step.

Initial void ratio: $e_o = wG_s = 0.62 \times 2.7 = 1.67$

$$\text{Equation (6.6):} \quad e = e_o - \frac{\Delta z}{H_o}(1 + e_o) = 1.67 - \frac{\Delta z}{20}(1 + 1.67)$$
$$= 1.67 - 13.35 \times 10^{-2}\Delta z$$

The void ratio for each load step is shown in the table below.

σ'_z (kPa)	15	30	60	120	240	480
Void ratio	1.66	1.65	1.64	1.52	1.38	1.25

A plot of $e - \log \sigma'_z$ versus e is shown in Fig. E6.14c.

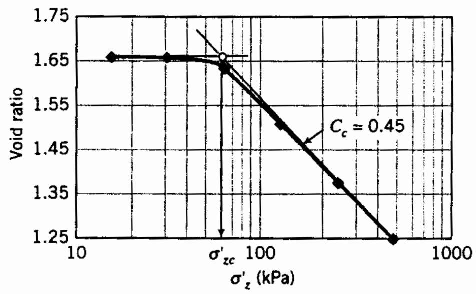

FIGURE E6.14c

Step 3: Determine σ'_{zc} and C_c.
Follow the procedures in Section 6.7 to find σ'_{zc}.

$$\sigma'_{zc} = 60\,\text{kPa}$$
$$C_c = \frac{1.52 - 1.25}{\log\left(\frac{480}{120}\right)} = 0.45$$

Step 4: Calculate σ'_{zo}.

$$\text{Clay:} \quad \gamma_{sat} = \frac{G_s + e_o}{1 + e_o}\gamma_w = \left(\frac{2.7 + 1.67}{1 + 1.67}\right)9.8 = 16\,\text{kN/m}^3$$
$$\sigma'_{zo} = (18 \times 2) + (19.4 - 9.8)1 + (16 - 9.8)1.5 = 54.9\,\text{kPa}$$

Step 5: Calculate settlement.

$$\text{OCR} = \frac{\sigma'_{zc}}{\sigma'_{zo}} = \frac{60}{54.9} = 1.1$$

For practical purpose, the OCR is very close to 1; that is, $\sigma'_z \approx \sigma'_{zc}$. Therefore, the soil is normally consolidated. Also, inspection of the e versus $\log \sigma'_z$ curve shows that C_r is approximately zero, which lends further support to the assumption that the soil is normally consolidated.

$$\rho_{pc} = \frac{H_o}{1 + 1.67}0.45\left(\frac{\sigma'_{zo} + \Delta\sigma_z}{\sigma'_{zo}}\right) = 0.17H_o \log\left(\frac{\sigma'_{zo} + \Delta\sigma_z}{\sigma'_{zo}}\right)$$

Divide the clay layer into three sublayers of 1.0 m thick and compute the settlement for each sublayer. The primary consolidation settlement is the sum of the settlement of each sublayer. The vertical stress increase in the fine-grained soil layer is

$$90 - \left(\frac{90 - 75}{3}\right)z = 90 - 5z$$

where z is the depth below the top of the layer. Calculate the vertical stress increase at the center of each sublayer and then the settlement from the above equation. The table below summarizes the computation.

Layer	z (m)	σ'_{zo} at center of sublayer (kPa)	$\Delta\sigma_z$	$\sigma'_{zo} + \Delta\sigma_z$ (kPa)	ρ_{pc} (mm)
1	0.5	48.7	87.5	136.2	75.9
2	1.5	54.9	82.5	137.4	67.7
3	2.5	61.1	77.5	138.6	60.5
				Total	204.1

Alternatively, by considering the whole fine-grained soil layer and taking the average vertical stress increment, we obtain

$$\rho_{pc} = \frac{3000}{1 + 1.67} 0.45 \log\left(\frac{137.4}{54.9}\right) = 201.4\,\text{mm}$$

In general, the former approach is more accurate for thick layers.

Step 6: Calculate settlement–time values.

$$C_v = 59.3 \times 10^{-6} \times 60 \times 24 = 85392 \times 10^{-6}\,\text{m}^2/\text{day}$$

$$t = \frac{T_v H_{dr}^2}{C_v} = \frac{T_v \times \left(\frac{3}{2}\right)^2}{85392 \times 10^{-6}} = 26.3\,T_v\,\text{days}$$

The calculation of settlement at discrete times is shown in the table below and the data are plotted in Fig. E6.14d.

U (%)	T_v	Settlement (mm) $\rho_{pc} \times \frac{U}{100}$	$t = 26.3\,T_v$ (days)
10	0.008	20.4	0.2
20	0.031	20.8	0.8
30	0.071	61.3	1.9
40	0.126	81.6	3.3
50	0.197	102.1	5.2
60	0.287	122.5	7.6
70	0.403	142.9	10.6
80	0.567	163.3	14.9
90	0.848	183.7	22.3

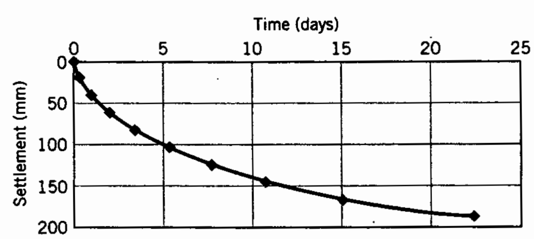

FIGURE E6.14d.

■

the soil is normally
ɔproximately zero,
ɔlidated.

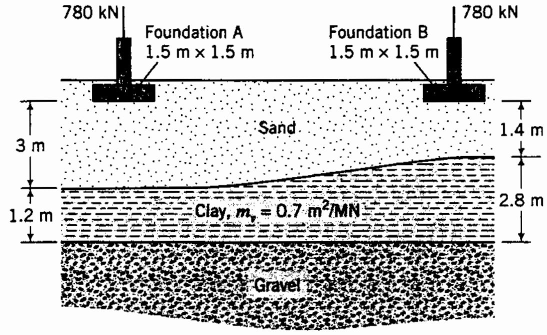

FIGURE E6.16

Strategy Determine the settlement under each foundation and then calculate the differential settlement. Since you know the differential settlement that occurs over a two year period after construction, you can find the degree of consolidation and then use this information to calculate the expected time for the doors to become jammed.

Solution 6.16

Step 1: Calculate the vertical stress increase at the center of the clay layer under each foundation.

You can use Eq. (5.81) or Eq. (5.85) or, for a quick estimate, Eq. (5.95) to determine the stress increase under the foundations. Let's use the approximate method, Eq. (5.95).

$$\Delta\sigma_z = \frac{P}{(B+z)(L+z)}$$

$$(\Delta\sigma_z)_A = \frac{780}{(1.5+3.6)(1.5+3.6)} = 30\,\text{kPa}$$

$$(\Delta\sigma_z)_B = \frac{780}{(1.5+2.8)(1.5+2.8)} = 42.2\,\text{kPa}$$

Note: For a more accurate value of $\Delta\sigma_z$ you should use the vertical stress increase due to surface loads on multilayered soils (Poulos and Davis, 1974).

Step 2: Calculate the primary consolidation settlement.

Use Eq. (6.18), $\rho_{pc} = H_o m_v \Delta\sigma$, to calculate the primary consolidation settlement.

$$(\rho_{pc})_A = 1.2 \times 0.7 \times 10^{-3} \times 30 = 25.2 \times 10^{-3}\,\text{m} = 25.2\,\text{mm}$$

$$(\rho_{pc})_B = 2.8 \times 0.7 \times 10^{-3} \times 42.2 = 82.7 \times 10^{-3}\,\text{m} = 82.7\,\text{mm}$$

Step 3: Calculate the differential settlement.

$$\text{Differential settlement:} \quad \delta = 82.7 - 25.2 = 57.5\,\text{min}$$

Step 4: Calculate the time for 24 mm differential settlement to occur.

Current differential settlement: $\delta_c = 10\,\text{mm}$

Degree of consolidation: $U = \dfrac{\delta_c}{\delta} = \dfrac{10}{57.5} = 0.17$

From Eq. (6.34): $T_v = \dfrac{4}{\pi}U^2 = \dfrac{4}{\pi} \times 0.17^2 = 0.037$

From Eq. (6.31): $C_v = \dfrac{T_v H_{dr}^2}{t} = \dfrac{0.037 \times (2.8/2)^2}{2} = 0.036\,\text{m}^2/\text{yr}$

For 24 mm differential settlement: $U = \dfrac{24}{57.5} = 0.42, \quad T_v = \dfrac{4}{\pi} \times 0.42^2 = 0.225$

From Eq. (6.31): $t = \dfrac{T_v H_{dr}^2}{C_v} = \dfrac{0.225 \times (2.8/2)^2}{0.036} = 12.25\,\text{years}$

Therefore, in the next 10.25 years, the total differential settlement would be 24 mm. ■

EXERCISES

For all problems, assume $G_s = 2.7$ unless otherwise stated.

Theory

6.1 A clay soil of thickness H is allowed to drain on the top boundary through a thin sand layer. A vertical stress of σ was applied to the clay. The excess porewater pressure distribution was linear in the soil layer with a value of u_t at the top boundary and u_b $(u_b > u_t)$ at the bottom boundary. The excess porewater pressure at the top boundary was not zero because the sand layer was partially blocked. Derive an equation for the excess porewater pressure distribution with soil thickness and time.

6.2 A soil layer of thickness H_o has only single drainage through the top boundary. The excess porewater pressure distribution when a vertical stress, σ, is applied varies parabolically with a value of zero at the top boundary and u_b at the bottom boundary. Show that

$$C_v = \frac{H_o^2}{2u_b}\frac{d\sigma'}{dt} \quad \text{and} \quad k_z = \frac{\gamma_w H_o}{2u_b}\frac{dH_o}{dt}$$

6.3 Show that, for a linear elastic soil,

$$m_v = \frac{(1+\nu')(1-2\nu')}{E'(1-\nu')}$$

6.4 Show that, if an overconsolidated soil behaves like a linear elastic material,

$$K_o^{oc} = (\text{OCR})K_o^{nc} - \frac{\nu'}{1-\nu'}(\text{OCR}-1)$$

6.5 The excess porewater pressure distribution in a 10 m thick clay varies linearly from 100 kPa at the top to 10 kPa at the bottom of the layer when a vertical stress was applied. Assuming drainage only at the top of the clay layer, determine the excess porewater pressure in 1 year's time using the finite difference method if $C_v = 1.5\,m^2/\text{yr}$.

6.6 At a depth of 4 m in a clay deposit, the overconsolidation ratio is 3.0. Plot the variation of overconsolidation ratio and water content with depth for this deposit up to a depth of 15 m. The recompression index is $C_r = 0.05$ and the water content at 4 m is 32%.