6.4 CALCULATION OF PRIMARY CONSOLIDATION SETTLEMENT

Interactive Concept Learning and Self-Assessment

Access Chapter 6, Section 6.4 on the CD to learn how to calculate one-dimensional consolidation settlement. Take Quiz 6.4 to assess your understanding and get problem-solving practice.

6.4.1 Effects of Unloading/Reloading of a Soil Sample Taken from the Field

Let us consider a soil sample that we wish to take from the field at a depth z (Fig. 6.4a). We will assume that the groundwater level is at the surface. The current vertical effective stress or overburden effective stress is

$$\sigma'_{zo} = (\gamma_{sat} - \gamma_w)z = \gamma' z$$

and the current void ratio can be found from γ_{sat} using Eq. 3.11. On a plot of e versus σ'_z (log scale), the current vertical effective stress can be represented as A as depicted in Fig. 6.4b.

To obtain a sample, we would have to make a borehole and remove the soil above it. The act of removing the soil and extracting the sample reduces the total stress to zero; that is, we have fully unloaded the soil. From the principle of effective stress [Eq. (5.39)], $\sigma'_z = -\Delta u$. Since σ' cannot be negative—that is, soil cannot sustain tension—the porewater pressure must be negative. As the porewater pressure dissipates with time, volume changes (swelling) occur. Using the basic concepts of consolidation described in Section 6.3, the sample will follow an unloading path AB (Fig. 6.4b). The point B does not correspond to zero effective stress because we cannot represent zero on a logarithm scale. However, the effective stress level at the start of the logarithmic scale is assumed to be small (≈ 0). If we were to reload our soil sample, the reloading path followed depends on the OCR. If OCR = 1 (normally consolidated soil), the path followed during reloading would be BCD (Fig. 6.4b). The average slope of ABC is C_r. Once σ'_{zo} is exceeded, the soil will follow the normal consolidation line, CD, of slope C_c. If the soil were overconsolidated, OCR > 1, the reloading path followed would be BEF because we have to reload the soil beyond its preconsolidation stress before it behaves like a

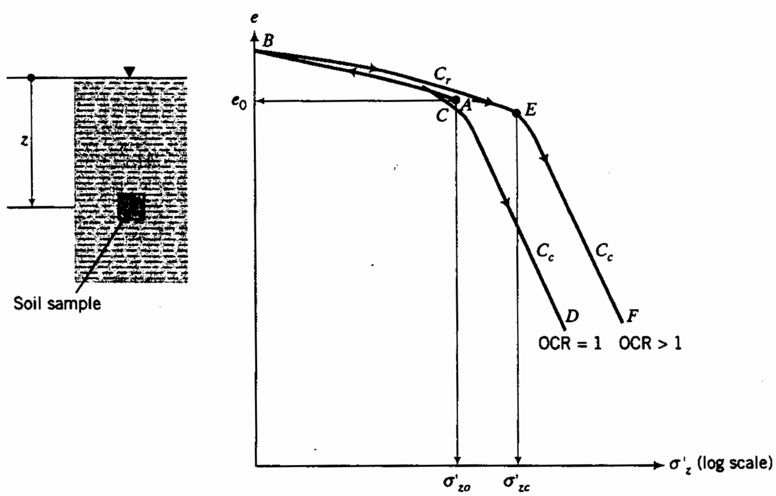


FIGURE 6.4 (a) Soil sample at a depth z below ground surface. (b) Expected one-dimensional response.

normally consolidated soil. The average slope of ABE is C_r and the slope of EF is C_c. The point E marks the preconsolidation stress. Later in this chapter, we will determine the position of E from laboratory tests (Section 6.7).

6.4.2 Primary Consolidation Settlement of Normally Consolidated Fine-Grained Soils

Let us consider a site consisting of a normally consolidated soil on which we wish to construct a building. We will assume that the increase in vertical stress due to the building at depth z, where we took our soil sample, is $\Delta\sigma_z$. (Recall that you can find $\Delta\sigma_z$ using the methods described in Section 5.11.) The final vertical stress is

$$\boxed{\sigma'_{\text{fin}} = \sigma'_{zo} + \Delta\sigma_z}$$

The increase in vertical stress will cause the soil to settle following the NCL and the primary consolidation settlement is

$$\boxed{\rho_{pc} = H_o \frac{\Delta e}{1 + e_o} = \frac{H_o}{1 + e_o} C_c \log \frac{\sigma'_{\text{fin}}}{\sigma'_{zo}};\ \text{OCR} = 1} \tag{6.14}$$

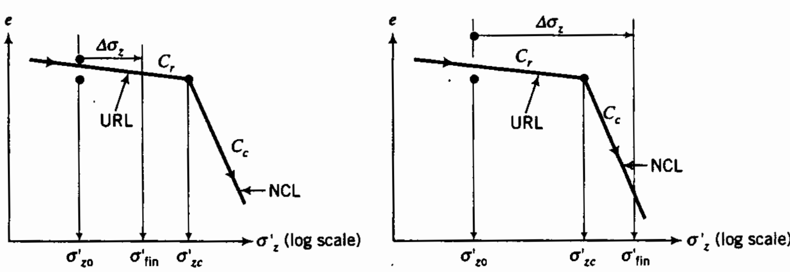
where $\Delta e = C_c \log(\sigma'_{\text{fin}}/\sigma'_{zo})$.

6.4.3 Primary Consolidation Settlement of Overconsolidated Fine-Grained Soils

If the soil is overconsolidated, we have to consider two cases depending on the magnitude of $\Delta\sigma_z$. We will approximate the curve in the σ'_z (log scale) versus e space as two straight lines, as shown in Fig. 6.5. In Case 1, the increase in $\Delta\sigma_z$ is such that $\sigma'_{\text{fin}} = \sigma'_{zo} + \Delta\sigma_z$ is less than σ'_{zc} (Fig. 6.5a). In this case, consolidation occurs along the URL and

$$\boxed{\rho_{pc} = \frac{H_o}{1 + e_o} C_r \log \frac{\sigma'_{\text{fin}}}{\sigma'_{zo}};\ \sigma'_{\text{fin}} < \sigma'_{zc}} \tag{6.15}$$

In Case 2, the increase in $\Delta\sigma_z$ is such that $\sigma'_{\text{fin}} = \sigma'_{zo} + \Delta\sigma_z$ is greater than σ'_{zc} (Fig. 6.5b). In this case, we have to consider two components of settlement—one along the URL and the other along the



(a) Case 1: $\sigma'_{zo} + \Delta\sigma_z < \sigma'_{zc}$ (b) Case 2: $\sigma'_{zo} + \Delta\sigma_z > \sigma'_{zc}$

FIGURE 6.5 Two cases to consider for calculating settlement of overconsolidated fine-grained soils.

NCL. The equation to use in Case 2 is

$$\rho_{pc} = \frac{H_o}{1+e_o}\left(C_r \log\frac{\sigma'_{zc}}{\sigma'_{zo}} + C_c \log\frac{\sigma'_{fin}}{\sigma'_{zc}}\right); \ \sigma'_{fin} > \sigma'_{zc} \qquad (6$$

or

$$\rho_{pc} = \frac{H_o}{1+e_o}\left[C_r \log(\text{OCR}) + C_c \log\frac{\sigma'_{fin}}{\sigma'_{zc}}\right]; \ \sigma'_{fin} > \sigma'_{zc} \qquad (6$$

6.4.4 Procedure to Calculate Primary Consolidation Settlement

The procedure to calculate primary consolidation settlement is as follows:

1. Calculate the current vertical effective stress (σ'_{zo}) and the current void ratio (e_o) at the cente the soil layer for which settlement is required.
2. Calculate the applied vertical stress increase ($\Delta\sigma_z$) at the center of the soil layer using appropriate method in Section 5.11.
3. Calculate the final vertical effective stress $\sigma'_{fin} = \sigma'_{zo} + \Delta\sigma_z$.
4. Calculate the primary consolidation settlement.
   (a) If the soil is normally consolidated (OCR = 1), the primary consolidation settlement is

   $$\rho_{pc} = \frac{H_o}{1+e_o} C_c \log\frac{\sigma'_{fin}}{\sigma'_{zo}}$$

   (b) If the soil is overconsolidated and $\sigma'_{fin} < \sigma'_{zc}$, the primary consolidation settlement is

   $$\rho_{pc} = \frac{H_o}{1+e_o} C_r \log\frac{\sigma'_{fin}}{\sigma'_{zo}}$$

   (c) If the soil is overconsolidated and $\sigma'_{fin} > \sigma'_{zc}$, the primary consolidation settlement is

   $$\rho_{pc} = \frac{H_o}{1+e_o}\left(C_r \log(\text{OCR}) + C_c \log\frac{\sigma'_{fin}}{\sigma'_{zc}}\right)$$

   where H_o is the thickness of the soil layer.

You can also calculate the primary consolidation settlement using m_v. However, unlike C_c, wh is constant, m_v varies with stress levels. You should compute an average value of m_v over the str range σ'_{zo} to σ'_{fin}. The primary consolidation settlement, using m_v, is

$$\rho_{pc} = H_o m_v \Delta\sigma_z \qquad (6.$$

The advantage of using Eq. (6.18) is that m_v is readily determined from displacement data consolidation tests; you do not have to calculate void ratio changes from the test data as requir to determine C_c.

6.4.5 Thick Soil Layers

For better accuracy, when dealing with thick layers ($H_o > 2\,m$), you should divide the soil layer in sublayers (about two to five sublayers) and find the settlement for each sublayer. Add up t settlement of each sublayer to find the total primary consolidation settlement. You must rememb that the value of H_o in the primary consolidation equations is the thickness of the sublayer. A alternative method is to use a harmonic mean value of the vertical stress increase for the sublayers the equations for primary consolidation settlement. The harmonic mean stress increase is

$$\Delta\sigma_z = \frac{n(\Delta\sigma_z)_1 + (n-1)(\Delta\sigma_z)_2 + (n-2)(\Delta\sigma_z)_3 + \cdots + (\Delta\sigma_z)_n}{n + (n-1) + (n-2) + \cdots + 1} \tag{6.19}$$

where n is the number of sublayers and the subscripts 1, 2, etc., mean the first (top) layer, the second layer, and so on. The advantage of using the harmonic mean is that the settlement is skewed in favor of the upper part of the soil layer. You should recall from Chapter 5 that the increase in vertical stress decreases with depth. Therefore, the primary consolidation settlement of the upper portion of the soil layer can be expected to be more than the lower portion because the upper portion of the soil layer is subjected to higher vertical stress increases.

EXAMPLE 6.1 *Consolidation Settlement of a Normally Consolidated Clay*

Interactive Problem Solving
Access Chapter 6, click Problem Solver on the sidebar menu and then click on Example 6.1 to interactively solve problems dealing with one-dimensional consolidation settlement.

The soil profile at a site for a proposed office building consists of a layer of fine sand 10.4 m thick above a layer of soft normally consolidated clay 2 m thick. Below the soft clay is a deposit of coarse sand. The groundwater table was observed at 3 m below ground level. The void ratio of the sand is 0.76 and the water content of the clay is 43%. The building will impose a vertical stress increase of 140 kPa at the middle of the clay layer. Estimate the primary consolidation settlement of the clay. Assume the soil above the water table to be saturated, $C_c = 0.3$ and $G_s = 2.7$.

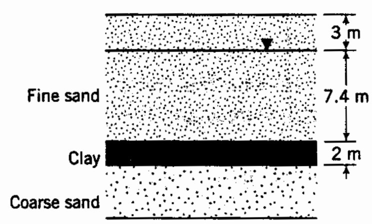


FIGURE E6.1

Strategy You should write down what is given or known and draw a diagram of the soil profile (see Fig. E6.1). In this problem, you are given the stratigraphy, groundwater level, vertical stress increase, and the following soil parameters and soil condition:

$$e_o \text{ (for sand)} = 0.76; w \text{ (for clay)} = 43\%$$
$$H_o = 2\,\text{m}, \Delta\sigma_2 = 140\,\text{kPa}, C_c = 0.3, G_s = 2.7$$

Since you are given a normally consolidated clay, the primary consolidation settlement response of the soil will follow path ABE (Fig. 6.3). The appropriate equation to use is Eq. (6.14).

Solution 6.1

Step 1: Calculate σ'_{zo} and e_o at the center of the clay layer.

$$\text{Sand: } \gamma_{sat} = \left(\frac{G_s + e}{1+e}\right)\gamma_w = \left(\frac{2.7 + 0.76}{1 + 0.76}\right)9.8 = 19.3\,\text{kN/m}^3$$

$$\gamma' = \left(\frac{G_s - 1}{1+e}\right)\gamma_w = \left(\frac{2.7 - 1}{1 + 0.76}\right)9.8 = 9.5\,\text{kN/m}^3$$

$$\text{or } \gamma' = \gamma_{sat} - \gamma_w = 19.3 - 9.8 = 9.5\,\text{kN/m}^3$$

$$\text{Clay: } e_o = wG_s = 2.7 \times 0.43 = 1.16;$$

$$\gamma' = \left(\frac{G_s - 1}{1+e}\right)\gamma_w = \left(\frac{2.7 - 1}{1 + 1.16}\right)9.8 = 7.7\,\text{kN/m}^3$$

The vertical effective stress at the mid-depth of the clay layer is

$$\sigma'_{zo} = (19.3 \times 3) + (9.5 \times 7.4) + (7.7 \times 1) = 135.9\,\text{kPa}$$

Step 2: Calculate the increase of stress at the mid-depth of the clay layer. You do not need to calculate $\Delta\sigma_z$ for this problem. It is given as $\Delta\sigma_z = 140\,\text{kPa}$.

Step 3: Calculate σ'_{fin}.

$$\sigma'_{fin} = \sigma'_{zo} + \Delta\sigma_z = 135.9 + 140 = 275.9\,\text{kPa}$$

Step 4: Calculate the primary consolidation settlement.

$$\rho_{pc} = \frac{H_o}{1+e_o} C_c \log\frac{\sigma'_{fin}}{\sigma'_{zo}} = \frac{2}{1+1.16} \times 0.3 \log\frac{275.9}{135.9} = 0.085\,\text{m} = 85\,\text{mm}$$ ■

EXAMPLE 6.2 ***Consolidation Settlement of an Overconsolidated Clay***

Assume the same soil stratigraphy as in Example 6.1. But now the clay is overconsolidated with an OCR = 2.5, $w = 38\%$, and $C_r = 0.05$. All other soil values given in Example 6.1 remain unchanged. Determine the primary consolidation settlement of the clay.

Strategy Since the soil is overconsolidated, you will have to check whether the preconsolidation stress is less than or greater than the sum of the current vertical effective stress and the applied vertical stress at the center of the clay. This check will determine the appropriate equation to use. In this problem, the unit weight of the sand is unchanged but the clay has changed.

Solution 6.2

Step 1: Calculate σ'_{zo} and e_o at mid-depth of the clay layer.
You should note that this settlement is small compared with the settlement obtained in Example 6.1.

$$\text{Clay: } e_o = wG_s = 0.38 \times 2.7 = 1.03$$

$$\gamma' = \left(\frac{G_s - 1}{1+e}\right)\gamma_w = \left(\frac{2.7 - 1}{1 + 1.03}\right)9.8 = 8.2\,\text{kN/m}^3$$

$$\sigma'_{zo} = (19.3 \times 3) + (9.5 \times 7.4) + (8.2 \times 1) = 136.4\,\text{kPa}$$

(note that the increase in vertical effective stress from the unit weight change in this overconsolidated clay is very small)

Step 2: Calculate the preconsolidation stress.

$$\sigma'_{zc} = 136.4 \times 2.5 = 341\,\text{kPa}$$

Step 3: Calculate σ'_{fin}

$$\sigma'_{fin} = \sigma'_{zo} + \Delta\sigma_z = 136.4 + 140 = 276.4\,\text{kPa}$$

Step 4: Check if σ'_{fin} is less than or greater than σ'_{zc}.

$$(\sigma'_{fin} = 276.4\,\text{kPa}) < (\sigma'_{zc} = 341\,\text{kPa})$$

Therefore, use Eq. (6.15).

Step 5: Calculate the total primary consolidation settlement.

$$\rho_{pc} = \frac{H_o}{1+e_o} C_r \log\frac{\sigma'_{fin}}{\sigma'_{zo}} = \frac{2}{1+1.03} \times 0.05 \log\frac{276.4}{136.4} = 0.015\,\text{m} = 15\,\text{mm}$$ ■

EXAMPLE 6.3 *Consolidation Settlement of an Overconsolidated Clay*

Assume the same soil stratigraphy and soil parameters as in Example 6.2 except that the clay has an overconsolidation ratio of 1.5. Determine the primary consolidation settlement of the clay.

Strategy Since the soil is overconsolidated, you will have to check whether the preconsolidation stress is less than or greater than the sum of the current vertical effective stress and the applied vertical stress at the center of the clay. This check will determine the appropriate equation to use.

Solution 6.3

Step 1: Calculate σ'_{zo} and e_o.
From Example 6.2, $\sigma'_{zo} = 136.4\,\text{kPa}$.

Step 2: Calculate the preconsolidation stress.

$$\sigma'_{zc} = 136.4 \times 1.5 = 204.6\,\text{kPa}$$

Step 3: Calculate σ'_{fin}.

$$\sigma'_{fin} = \sigma'_{zo} + \Delta\sigma_z = 136.4 + 140 = 276.4\,\text{kPa}$$

Step 4: Check if σ'_{fin} is less than or greater than σ'_{zc}.

$$(\sigma'_{fin} = 276.4\,\text{kPa}) > (\sigma'_{zc} = 204.6\,\text{kPa})$$

Therefore, use either Eq. (6.16) or (6.17).

Step 5: Calculate the total primary consolidation settlement.

$$\rho_{pc} = \frac{H_o}{1+e_o}\left\{C_r \log\frac{\sigma'_{zc}}{\sigma'_{zo}} + C_c \log\frac{\sigma'_{fin}}{\sigma'_{zc}}\right\} = \frac{2}{1+1.03}$$

$$\times \left(0.05 \log\frac{204.6}{136.4} + 0.3 \log\frac{276.4}{204.6}\right) = 0.047\,\text{m} = 47\,\text{mm}$$

or

$$\rho_{pc} = \frac{H_o}{1+e_o}\left\{C_r \log(\text{OCR}) + C_c \log\frac{\sigma'_{fin}}{\sigma'_{zc}}\right\} = \frac{1}{1+1.03}$$

$$\times \left(0.05 \log 1.5 + 0.3 \log\frac{276.4}{204.6}\right) = 0.047\,\text{m} = 47\,\text{mm}$$ ■

EXAMPLE 6.4 *Consolidation Settlement Using m_v*

A vertical section through a building foundation at a site is shown in Fig. E6.4. The average modulus volume compressibility of the clay is $m_v = 5 \times 10^{-5}\,m^2/kN$. Determine the primary consolidatic settlement.

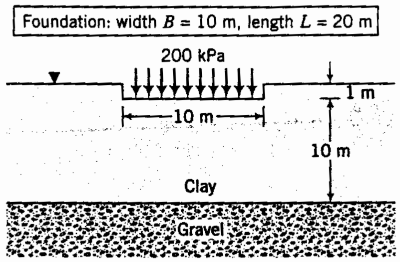


FIGURE E6.4

Strategy To find the primary consolidation settlement, you need to know the vertical stress increase in the clay layer from the building load. Since the clay layer is finite, we will have to use the vertical stress influence values in Appendix C. If we assume a rough base, we can use the influence values specified by Milovic and Tournier (1971) or if we assume a smooth base we can use the values specified by Sovinc (1961). The clay layer is 10 m thick, so it is best to subdivide the clay layer into sublayers ≤ 2 m thick.

Solution 6.4

Step 1: Find the vertical stress increase at the center of the clay layer below the foundation. Divide the clay layer into five sublayers, each of thickness 2 m—that is, $H_o = 2\,m$. Find the vertical stress increase at the middle of each sublayer under the center of the rectangular foundation. Assume a rough base and use Table C1 (Appendix C).

$$B = 10\,m, \quad L = 20\,m, \quad \frac{L}{B} = 2, \quad q_s = 200\,kPa$$

Layer	z (m)	$\frac{z}{B}$	I_{zp}	$\Delta\sigma_z = I_{zp} q_s$ (kPa)
1	1	0.1	0.992	198.4
2	3	0.3	0.951	190.2
3	5	0.5	0.876	175.2
4	7	0.7	0.781	156.2
5	9	0.9	0.686	137.2

Step 2: Calculate the primary consolidation settlement. Use Eq. (6.18).

$$\rho_{pc} = \sum_{i=1}^{n} (H_o m_v \Delta\sigma_z)_i = 2 \times 5 \times 10^{-5} \times (198.4 + 190.2 + 175.2 + 156.2 + 137.2) = 0.086\,m = 86\,mm$$

Alternatively: Use the harmonic mean value of $\Delta\sigma_z$ with $n = 5$; that is, Eq. (6.19)

$$\Delta\sigma_z = \frac{5(198.4) + 4(190.2) + 3(175.2) + 2(156.2) + 1(137.2)}{5+4+3+2+1} = 181.9\,\text{kPa}$$

$$\rho_{pc} = 10 \times 5 \times 10^{-5} \times 181.9 = 0.091\,\text{m} = 91\,\text{mm}$$

The greater settlement in this method results from the bias toward the top layer. ■

EXAMPLE 6.5 *Estimating OCR Variation with Depth*

A laboratory test on a saturated clay taken at a depth of 10 m below the ground surface gave the following results: $C_c = 0.3$, $C_r = 0.08$, OCR = 5, $w = 23\%$, and $G_s = 2.7$. The groundwater level is at the surface. Determine and plot the variation of water content and overconsolidation ratio with depth up to 50 m.

Strategy The overconsolidation state lies on the unloading/reloading line (Fig. 6.3), so you need to find an equation for this line using the data given. Identify what given data is relevant to finding the equation for the unloading/reloading line. Here you are given the slope, C_r, so you need to use the other data to find the complete question. You can find the coordinate of one point on the unloading/reloading line from the water content and the depth as shown in Step 1.

Solution 6.5

Step 1: Determine e_o and σ'_{zo}.

$$\gamma' = \left(\frac{G_s - 1}{1 + e}\right)\gamma_w = \left(\frac{27 - 1}{1 + 0.621}\right)9.8 = 10.3\,\text{kN/m}^3$$

$$e_o = G_s w = 2.7 \times 0.23 = 0.621$$

$$\sigma'_{zo} = \gamma' z = 10.3 \times 10 = 103\,\text{kPa}$$

Step 2: Determine the preconsolidation stress.

$$\sigma'_{zc} = \sigma'_{zo} \times \text{OCR} = 103 \times 5 = 515\,\text{kPa}$$

Step 3: Find the equation for the URL (slope BC in Fig. E6.5a).

$$e_B = e_o - C_r \log\frac{\sigma'_{zc}}{\sigma'_{zo}}$$

Therefore,

$$e_B = 0.621 - 0.08\log(5) = 0.565$$

Hence, the equation for the unloading/reloading line is

$$e = 0.565 + 0.08\log(\text{OCR}) \quad (1)$$

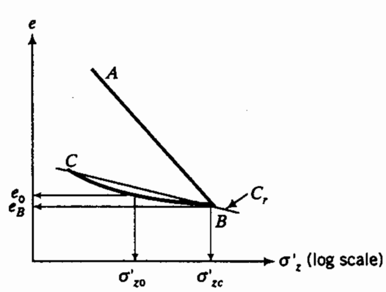


FIGURE E6.5a

Substituting $e = wG_s (G_s = 2.7)$ and $\text{OCR} = 515/\gamma' z$ ($\gamma' = 10.3\,\text{kN/m}^3$, z is depth) in Eq. (1) gives

$$w = 0.209 + 0.03 \log\left(\frac{50}{z}\right)$$

You can now substitute values of z from 1 to 50 and find w and substitute $e = wG_s$ in Eq. (1) to find the OCR. The table below shows the calculated values and the results, which are plotted in Fig. E6.5b.

Depth	w (%)	OCR
1	26.0	51.4
5	23.9	10
10	23.0	5
20	22.1	2.5
30	21.6	1.7
40	21.2	1.2
50	20.9	1.0

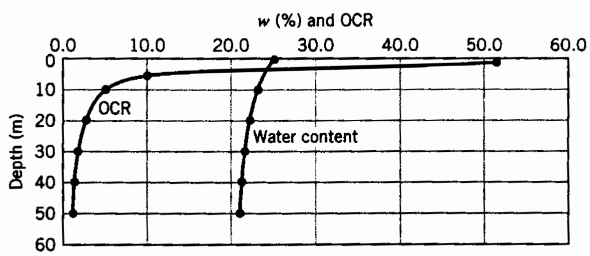


FIGURE E6.5b

You should note that the soil becomes normally consolidated as the depth increases. This is a characteristic of real soils. ■

What's next... So far, we have only considered how to determine the final primary consolidation settlement. This settlement might take months or years to occur, depending essentially on the permeability of the soil, the soil thickness, drainage conditions, and the magnitude of the applied stress. Geotechnical engineers have to know the magnitude of the final primary consolidation settlement and also the rate of settlement so that the settlement at any given time can be evaluated.

The next section deals with a theory to determine the settlement at any time. Several assumptions are made in developing this theory. However, you will see that many of the observations we made in Section 6.3 are well described by this theory.

6.5 ONE-DIMENSIONAL CONSOLIDATION THEORY

Interactive Concept Learning and Self-Assessment

Access Chapter 6, Section 6.5 on the CD to learn the derivation of the dimensional consolidation equation step by step. Take Quiz 6.5 to test your understanding on the interpretation of this equation.