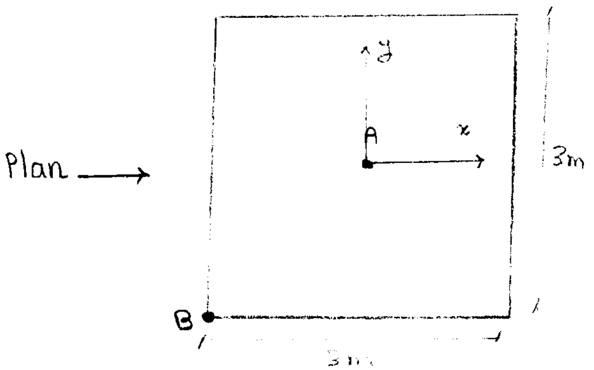
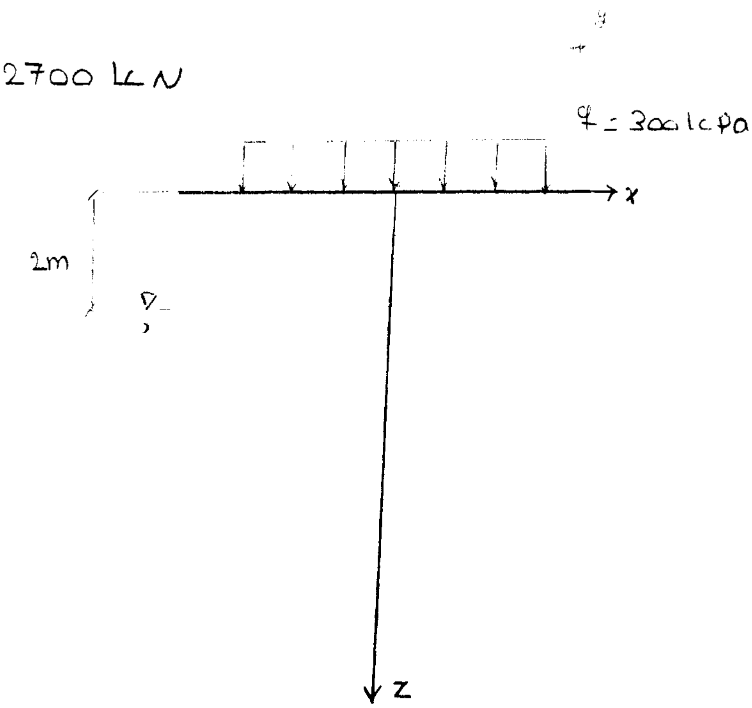
CivE 353- Solution to Assignment 7

Question 1

Square footing 3 x 3

$P = B^2 q = 9\,m^2 \times 300\,kN/m^2 = 2700\,kN$





Equation (6.17)

$$\Delta\sigma_z = \frac{3P}{2\pi} \frac{z^3}{(r^2 + z^2)^{5/2}}$$

Point A:

r = 0

$$\Longrightarrow \quad \boxed{\Delta\sigma_z = \frac{3 \times 2700}{2\pi z^2}}$$

Point B

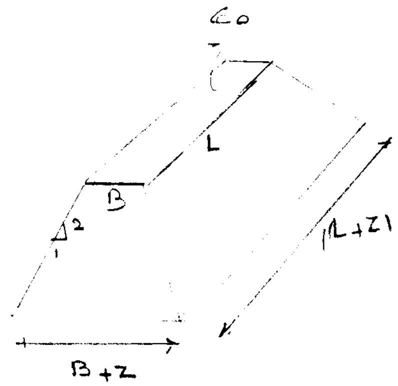
$r^2 = (1.5)^2 + (1.5)^2 = 4.5$

$$\Delta\sigma_z = \frac{3P}{2\pi} \frac{z^3}{(4.5 + z^2)^{5/2}}$$

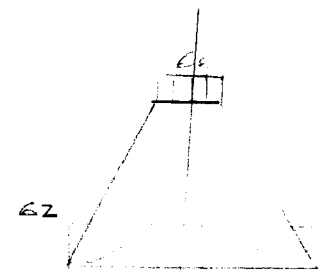
$$\boxed{= \frac{3 \times 2700}{2 \times \pi} \frac{z^3}{(4.5 + z^2)^{5/2}}}$$

b) Using Fadum chart

In this example

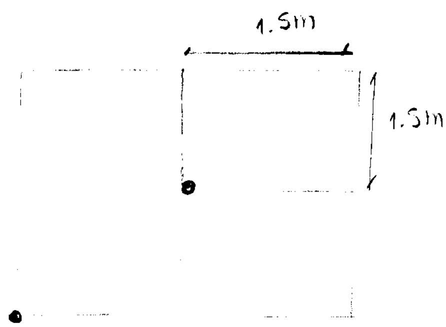


$$\sigma_z = \frac{\sigma_o B L}{(B+z)(L+z)}$$



In this question, $B = L = 3\,m$

$P = B^2 q = 2700\ kN$



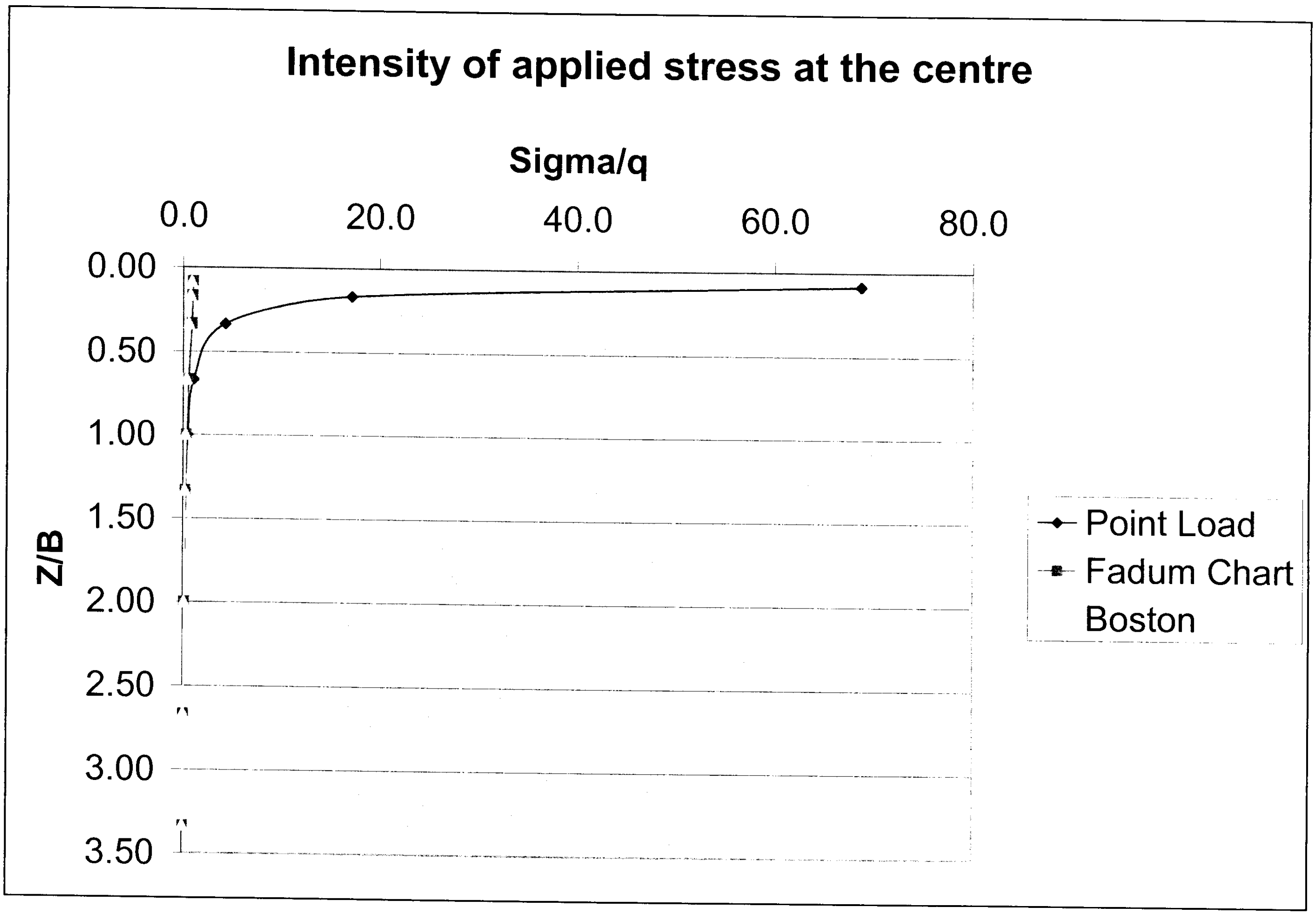
$B : 1.5$ → Fadum chart → $\frac{B}{z}$, $\frac{L}{z}$ → I_z

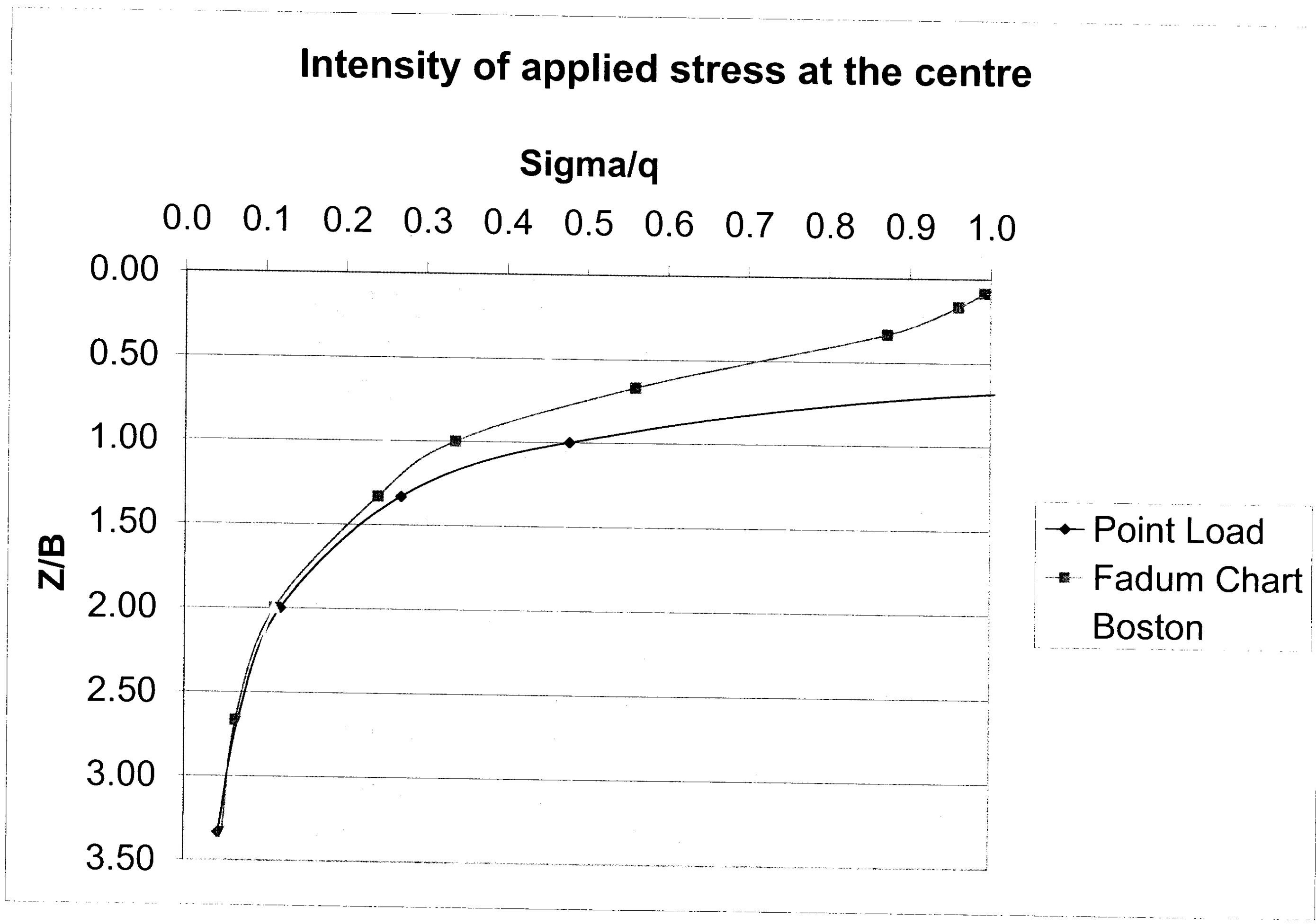
Question 1

$\sigma_z = 4q \times I_z = 4 \times 300 \times I_z$

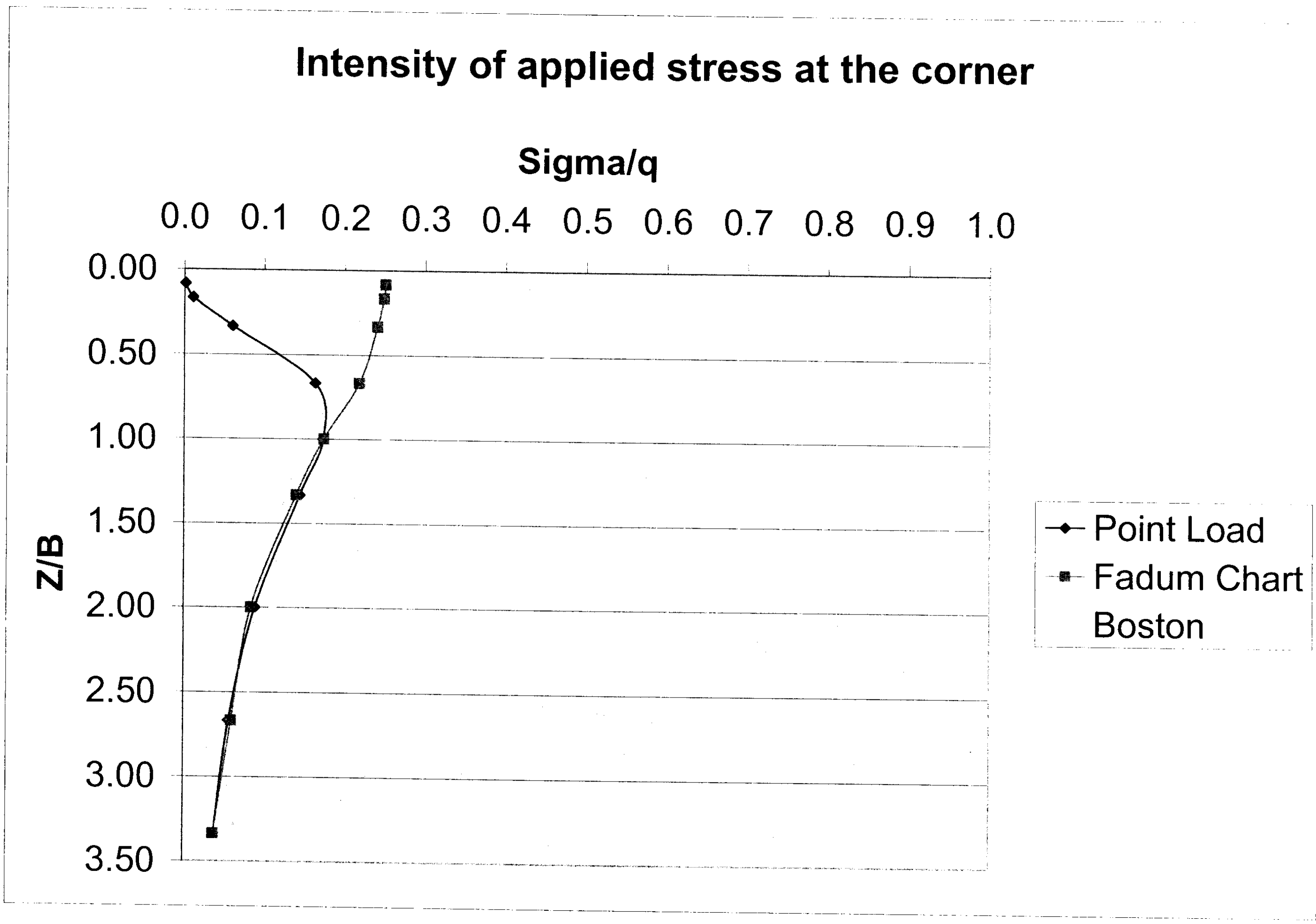
$$\sigma_z = \frac{300 \times 3 \times 3}{(3+z)(3+z)}$$

		Point Load		Fadum chart								Boston Approximation	
		Center	Corner	Corner				Center				Corner	Center
Z(m)	Z/B	sigma(KN/m^2)		m	n	I2	sigma	m	n	I2	sigma	sigma	sigma
0.25	0.08	20626.48	0.45	12.00	12.00	0.250	75.00	6.00	6.00	0.248	297.60	255.62	255.62
0.50	0.17	5156.62	3.28	6.00	6.00	0.248	74.40	3.00	3.00	0.240	288.00	220.41	220.41
1.00	0.33	1289.16	18.17	3.00	3.00	0.240	72.00	1.50	1.50	0.218	261.60	168.75	168.75
2.00	0.67	322.29	48.96	1.50	1.50	0.218	65.40	0.75	0.75	0.140	168.00	108.00	108.00
3.00	1.00	143.24	51.98	1.00	1.00	0.174	52.20	0.50	0.50	0.084	100.80	75.00	75.00
4.00	1.33	80.57	43.36	0.75	0.75	0.140	42.00	0.38	0.38	0.060	72.00	55.10	55.10
6.00	2.00	35.81	26.68	0.50	0.50	0.084	25.20	0.25	0.25	0.028	33.60	33.33	33.33
8.00	2.67	20.14	17.00	0.38	0.38	0.060	18.00	0.19	0.19	0.016	19.20	22.31	22.31
10.00	3.33	12.89	11.55	0.30	0.30	0.038	11.40	0.15	0.15	0.012	14.40	15.98	15.98


Intensity of applied stress at the centre
Sigma/q
0.0
20.0
40.0
60.0
80.0
0.00
0.50
1.00
1.50
2.00
2.50
3.00
3.50
Z/B
Point Load
Fadum Chart
Boston

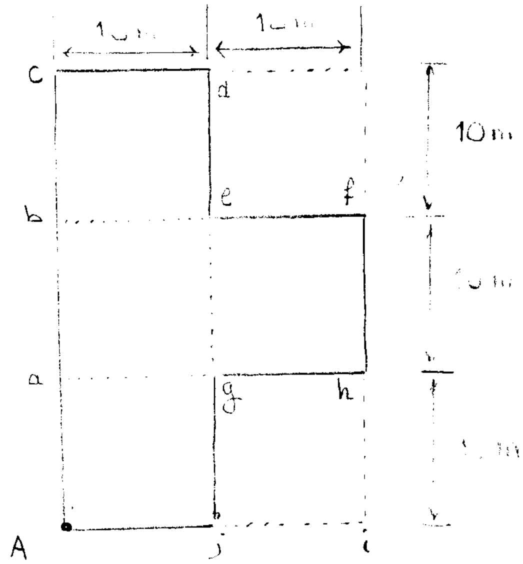


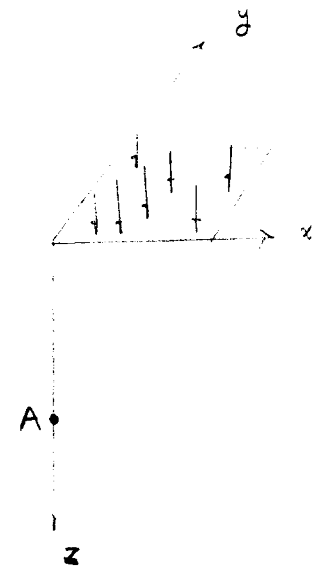
Intensity of applied stress at the centre
Sigma/q
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
0.00
0.50
1.00
1.50
2.00
2.50
3.00
3.50
Z/B
Point Load
Fadum Chart
Boston



Intensity of applied stress at the corner
Sigma/q
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
0.00
0.50
1.00
1.50
2.00
2.50
3.00
3.50
Z/B
Point Load
Fadum Chart
Boston

Question 2





Item \ Area	(1) + Abfi	(2) - Aahi	(3) + Acdj	(4) - Abej	(5) + Aagj
x	20	20	10	10	10
y	20	10	20	20	10
z	10	10	10	10	10
B/z = m	2	1	1	1	1
L/z : n	2	2	3	2	1
I_2	0.234	0.2	0.202	0.2	0.174
Δz					

\+ for loaded areas

\- for unloaded areas

+ Abfi − Aahj + Acdj − Abej + Aagj ⟶ result in the loaded foundation

$$\Delta\sigma = q\,[\,I_{2(1)} - I_{2(2)} + I_{2(3)} - I_{2(4)} + I_{2(5)}\,]$$

$$= 270\,[\,0.234 - 0.2 + 0.202 - 0.2 + 0.174\,]$$

$$= 270 \times 0.21 = 56.7 \text{ kpa}$$

52.5

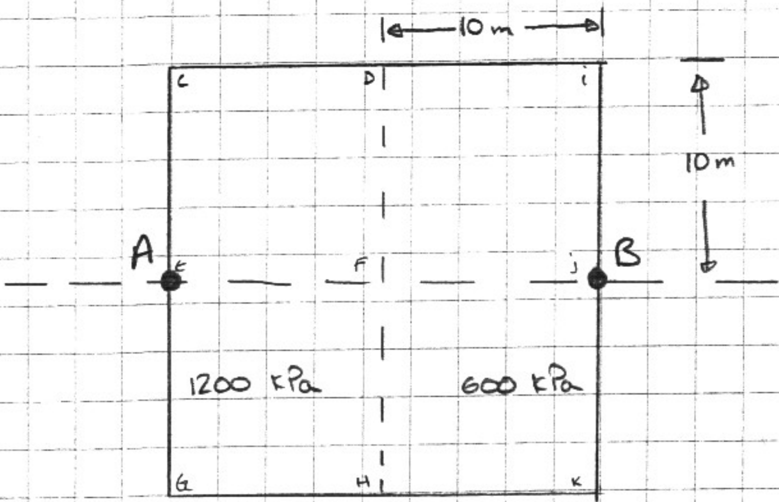
Scale 2.2 cm ⟶ 10 m

$\Delta\sigma$ = 43 × 0.005 × 270 = 59.4

= 58.05 kpa

Question 3

Determine the differential settlement of the pipe between points A & B.



* Need to find settlement of soil beneath the pipe (20m)
∴ choose mid point of that layer

Please note that this solution is for a pipe at 20m depth (not 10m).

$\sigma_o' = 20m(\gamma_{sat} - \gamma_w) = 20m(18 - 9.81)\ kN/m^3$
$= 163.8\ kPa$

* Find $\Delta\sigma_z'$ at midpoint of layer beneath pipe

		Area	q (kPa)	B	L	m	n	I_z
At A:	(1)	CEJi	600	10	20	0.5	1	0.123
	(2)	EGKJ	600	10	20	0.5	1	0.123
	(3)	CDEF	600	10	10	0.5	0.5	0.084
	(4)	EFGH	600	10	10	0.5	0.5	0.084
At B:	(1)	CEJi	1200	10	20	0.5	1	0.123
	(2)	EGKJ	1200	10	20	0.5	1	0.123
	(3)	DiJF	600	10	10	0.5	0.5	0.084
	(4)	FJHK	600	10	10	0.5	0.5	0.084

$$\therefore \quad \Delta\sigma'_{z_A} = q \cdot \left(\sum I_{z(i)}\right)$$

$$= 600\ kPa\ (0.123 \times 2 + 0.084 \times 2)$$

$$= 248.4\ kPa$$

$$\Delta\sigma'_{z_B} = q_1\left(I_{z(1)} + I_{z(2)}\right) - q_2\left(I_{z(3)} + I_{z(4)}\right)$$

$$= 1200\ kPa\ (0.123 \times 2) - 600\ kPa\ (0.083 \times 2)$$

$$= 195.6\ kPa$$

Because we have N/C clay we need to use the following eqn:

$$S_p = \frac{C_c \cdot H}{1 + e_o} \log\left(\frac{\sigma'_o + \Delta\sigma'_z}{\sigma'_o}\right)$$

At A:

$$S_{P_A} = \frac{0.35 \cdot (20\ m)}{1 + 1.0} \cdot \log\left(\frac{163.8\ kPa + 248.4\ kPa}{163.8\ kPa}\right)$$

$$= 1.402\ m$$

At B:

$$S_{P_B} = \frac{0.35\ (20m)}{1 + 1.0} \cdot \log\left(\frac{163.8 + 195.6}{163.8}\right)$$

$$= 1.194\ m$$

$\therefore$ the differential Settlement $= S_{P_A} - S_{P_B}$

$$= 0.208\ m.$$

Question 4

Estimate the magnitude of the ground surface settlement:

* Settlement will occur as a result of consolidation of the clay layer

* If the length + width of the fill are large (ie: extensive) the $\Delta\sigma_z$ is constant with depth

$$\therefore \Delta\sigma_z' = \gamma_{fill} \cdot H_{fill}$$

$$\gamma_{garabage} = 1830 \text{ g/cm}^3 \times 9.81 \text{ m/s}^2$$

$$= 17.95 \text{ kN/m}^3$$

$$\therefore \Delta\sigma_z' = 17.95 \text{ kN/m}^3 \cdot 30\text{m}$$
$$= 538.6 \text{ kPa}$$

$$\sigma_o' = \sum \gamma_i \cdot H_i - u$$

$$= 19.6 \text{ kN/m}^3 \cdot 5\text{m} + 22.2 \text{ kN/m}^3 \cdot 5\text{m} + 1\text{m} \cdot 19.3 \text{ kN/m}^3 - 6\text{m} \cdot 9.81 \text{ kN/m}^3$$

$$= 169.44 \text{ kPa}$$

$$\therefore \Delta\sigma_z' + \sigma_o' = 708.04 \text{ kPa}$$

$$OCR = \frac{\sigma_c'}{\sigma_o'} \qquad \therefore \sigma_c' = 2.1 \cdot (169.44 \text{ kPa})$$

$$\sigma_c' = 355.82 \text{ kPa}$$

$$\therefore \sigma_c' > \sigma_o' \text{ and } \sigma_o' + \Delta\sigma_z' > \sigma_c'$$

↳ use eqn 7.17

$$S_p = \frac{C_r \cdot H}{1+e_o} \log \frac{\sigma_c'}{\sigma_o'} + \frac{C_c \cdot H}{1+e_o} \log\left(\frac{\sigma_o' + \Delta\sigma_z'}{\sigma_c'}\right)$$

$e_o = \omega G_s$ when $S_r = 1$

$\therefore e_o = 0.48 \times 2.65$
$= 1.272$

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$\therefore S_p = \frac{0.03 \cdot 2m}{2.272} \cdot \log\left(\frac{355.82}{169.44}\right) + \frac{0.28 \cdot 2m}{2.272} \cdot \log\left(\frac{708.04}{355.82}\right)$

$= 0.082 \text{ m}$

$\therefore$ the ground surface settlement will be 8.2 cm

4b) Determine the time taken for consolidation:

$T_v = \frac{c_v \cdot t}{H_{dr}^2}$

$\therefore t = \frac{T_v \cdot H_{dr}^2}{c_v}$ * Assume settlement is complete at $U = 90\%$

* need to find T_v: 2 ways: - Equation
- Chart

Equation: for $u > 60\%$ $T_v = 1.781 - 0.933 \log(100 - u\%)$ (eqn 7.59)
$= 0.848$ for $U = 90\%$

Chart: $H_{dr} = 1m$ (water can drain in both directions)
$z = 0.5m$ (middle of the drainage path - an attempt at getting U_{avg} over entire drainge path)

$U = 90\%$

$\therefore$ from Figure 7.18 $T_v \simeq 0.85$

$\therefore t = \frac{T_v \cdot H_{dr}^2}{c_v} = \frac{0.85 \cdot (1m)^2}{0.95 \text{ m}^2/\text{yr}}$

$t = 0.89$ yrs $\therefore$ it will take 0.89 years

Question 5

How long will settlement take:

Consider the midpoint of the upper clay layer point A and the midpoint of the lower clay layer point B.

$\sigma'_{o_A} = 19.6\ kN/m^3 \cdot 5m + 1m(22.2\ kN/m^3) + 1m(19.3) - 2m(9.81)$

$= 119.88\ kPa$

$\sigma'_{o_B} = 119.88\ kPa + 1m(19.3\ kN/m^3) + 2m(22) + 3m(18.8) - 6m(9.81)$

$= 180.72\ kPa$

$\Delta\sigma'_A = 2m(9.81\ kN/m^3)$
$= 19.62\ kPa$

$\Delta\sigma'_B = 5m(9.81\ kN/m^3)$
$= 49.05\ kPa$

At layer A: $e_o = w G_s$ - N/c clay
$= 0.5 \cdot 2.65$
$= 1.325$

$\therefore S_p = \dfrac{C_c \cdot H}{1 + e_o} \log\left(\dfrac{\sigma'_o + \Delta\sigma'_A}{\sigma'_o}\right)$

$\therefore S_p = \dfrac{0.32 \cdot 2m}{1 + 1.325} \cdot \log\left(\dfrac{119.88 + 19.62}{119.88}\right)$

$= 0.018\ m$

At layer B: $e_o = w G_s$
$= 0.56 \times 2.68$
$= 1.5$

$OCR = 1.3 = \dfrac{\sigma'_c}{\sigma'_o}$

$\therefore \sigma'_c = 1.3 \cdot (180.72)$
$= 234.94\ kPa$

$\sigma'_o + \Delta\sigma'_B = 229.77$

$\therefore \sigma'_o < \sigma'_c$ & $\sigma'_o + \Delta\sigma'_B < \sigma'_c$

$$m_v = \frac{1}{1+e_o} \cdot \left(\frac{e_o - e_1}{\sigma_1' - \sigma_o'} \right)$$

$$= \frac{1}{H_o} \left(\frac{H_o - H_1}{\sigma_1' - \sigma_o'} \right)$$

Also $S_{P_B} = m_v \cdot \Delta\sigma' \cdot H$

$= 1.1 \; m^2/MN \cdot 49.05 \; kN/m^2 \cdot 6m$

$= 0.324 \; m$

$\therefore S_{P_{Total}} = S_{P_A} + S_{P_B}$

$= 0.018m + 0.324m$

$= 0.342 \; m$

$\therefore U = 50$ corresponds to 0.171 m

$U = 90$ corresponds to 0.308 m

Trial & error based on excel graph to make initial guess.

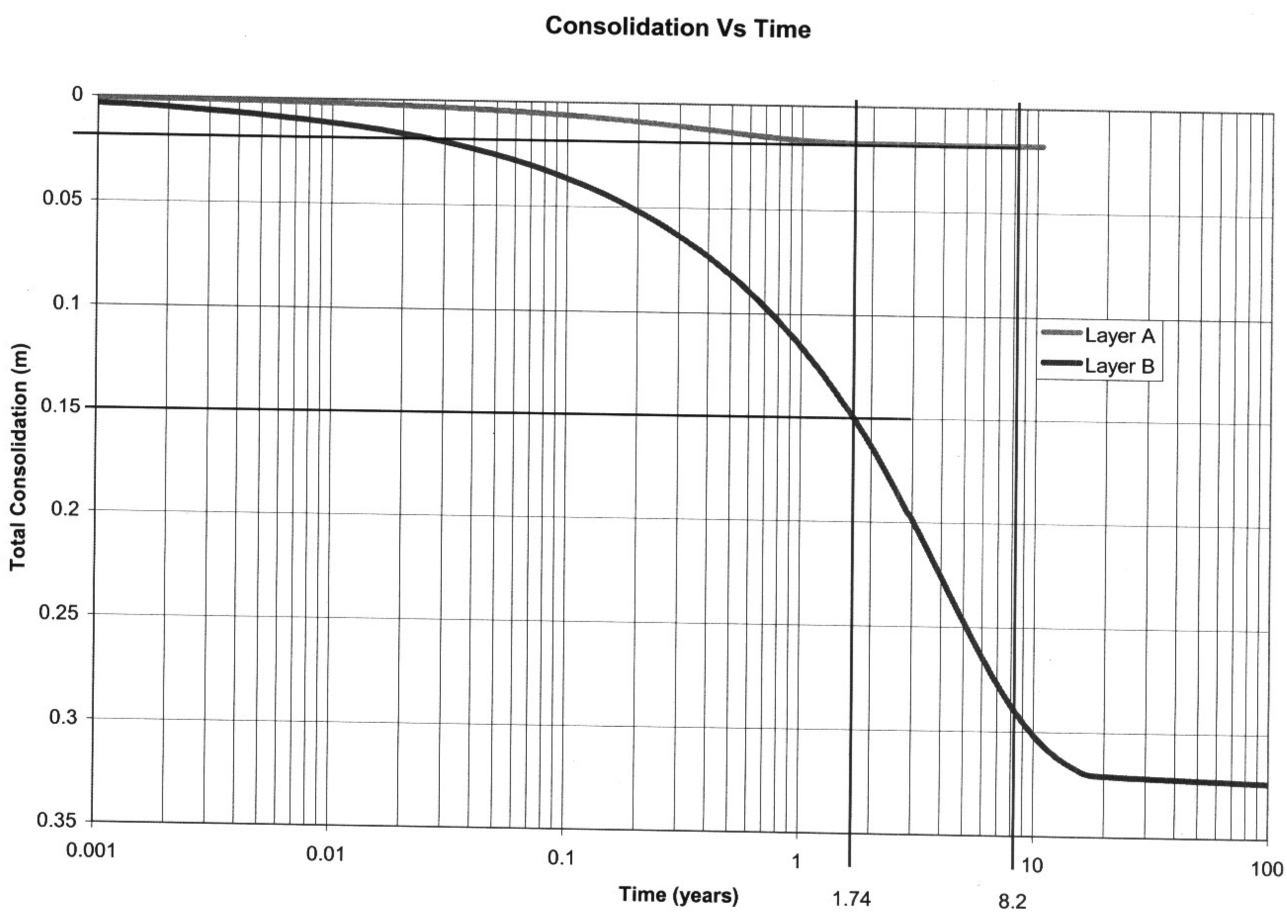
$$t_A = \frac{T_v \cdot H_{dr}^2}{C_v} = \frac{T_v \cdot (1m)^2}{0.95 \; m^2/yr}$$

$$t_B = \frac{T_v \cdot H_{dr}^2}{C_v} = \frac{T_v \cdot (3m)^2}{0.9 \; m^2/yr}$$

→ from trial & error & excell graph

$t \approx 1.74$ yrs for $U = 50\%$

$t \approx 8.2$ yrs for $U = 90\%$



Consolidation Vs Time
Total Consolidation (m)
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.001
0.01
0.1
1
10
100
1.74
8.2
Time (years)
Layer A
Layer B