1. Introduction
The increasing complexity and runtime of environmental models lead to the current situation where the calibration of all model variables or the estimation of all of their uncertainty is often computationally infeasible. Hence, techniques to determine the sensitivity of model variables are used to identify most important variables or model processes. While the examination of the convergence of calibration and uncertainty methods is state-of-the-art, the convergence of sensitivity analyses (SA) results is usually not checked. If any, bootstrapping of the sensitivity results is used to determine the reliability of estimated indexes. Bootstrapping, however, requires non-negligible implementation efforts and can also become computationally expensive in case of large model outputs and a high number of bootstraps. It also does not perform well for small sample sizes. We, therefore, present a Model Variable Augmentation (MVA) approach for improved interpretation of SA experiments. MVA is:
- method- and model independent
- computationally frugal
- applicable during or after the SA

2. Test functions & Experimental Setup
The method is tested using the methods of Sobol’ sensitivity indexes (Sobol’, 1993) and the PAWN indexes (Pianosi & Wagener, 2015). Different numbers of variable sets \( N_x \) (a proxy for number of model runs) were used, i.e. 10, 100, and 1000. To compare the results of MVA with standard approaches the indexes were also bootstrapped. The number of bootstrap samples was set to \( N_B = 1000 \). We employed 12 benchmark functions with different numbers of variables \( N_x \) to demonstrate the reliability of MVA.

3. Model Variable Augmentation MVA
- augment true model with artificial model variables \( z_0, z_1, \) and \( z_2 \) with known properties
- original model output \( f(x) \) is converted into \( y(x, z, c) \) where \( c \) is a correction factor such that \( \sigma_y^2 = \sigma_c^2 \)
- \( S \) describes the sensitivity of parameter \( i \)
- \( z_0 \) is dummy variable to check correctness of sensitivity method itself \( (S_0 = 0) \)
- \( z_1 \) and \( z_2 \) are variables which variances are controlled by an SA index specific parameter \( \Delta \) to check for sampling uncertainty \( (S_0 = S_\Delta) \)

   **Bootstrapping**
   Analyze mean \( \mu(B) \) and variance \( \sigma(B) \) of bootstrapped distribution \( D(B) \) of sensitivity indexes. Convergence, if rel. error is below \( \delta_c: \)
   \[
   \frac{\mu(B)}{\sigma(B)} < \delta_c \quad \forall x_i
   \]

   **MVA**
   Identify variables above certain threshold \( \delta_M \) as important:
   \[
   S_i > \delta_B
   \]

   Kolmogorov-Smirnov test to check if bootstrapped distributions of sensitivity indexes for two variables \( x_i \) and \( x_j \) are significantly different:
   \[
   H_0 : D_{S_i} = D_{S_j} \quad |S_i - S_j| \leq |S_0 - S_B|
   \]

4. Results

   **Fig. 1:** Results of a Sobol’ sensitivity analysis for the Oakley-O’Hagan test function using 10 variable sets and a confidence threshold \( \Delta \) of 0.2. (A) shows the Sobol’ sensitivity indexes with and without MVA as well as the true Sobol indexes. In (B) the individual indexes of all 1000 bootstraps and in (C) the ranking of the variables is depicted. (D) shows how many variables are enveloped by the augmented variables (red) and hence are not converged.

   **Fig. 2:** Results of a Sobol’ sensitivity analysis for the Oakley-O’Hagan test function using 100 variable sets and a confidence threshold \( \Delta \) of 0.2. The subplots are the same as in Figure 1.

   **Fig. 3:** Results of a PAWN sensitivity analysis for the Oakley-O’Hagan test function using 100 variable sets and a confidence threshold \( \Delta \) of 0.2. The individual subplots are the same as in Figure 1.

   **Fig. 4:** Sobol’ indexes of the dummy parameter \( z_1 \) and \( z_2 \) (gray box) relative to the other parameter sensitivities of the Oakley-O’Hagan test function using 10 variable sets and no bootstrap. Color indicates the parameter index \( i \).

   **Fig. 5:** PAWN indexes of the dummy parameters \( z_1 \) and \( z_2 \) (gray box) relative to the other parameter sensitivities of the Oakley-O’Hagan test function using 10 variable sets and no bootstrap.

   **Fig. 6:** Ratio of correctly determined informative variables using different confidence thresholds \( \Delta \) and different numbers of reference sets \( N_x \) used to estimate the Sobol’ sensitivities. The variable augmentation MVA is increasing the number of correctly identified informative variables in all experiments (compare squares and circles).

5. Conclusions
- MVA is computationally less expensive than bootstrapping since automatically computed during sens. estimation
- MVA indicates reliability of sens. estimates (Fig. 1 & 2)
- MVA allows for seamless check of certainty of SA results
- MVA identifies important variables more reliably than standard fixed-threshold method (Fig. 6)