Worked Cantilever Beam and Shear Flow Example

(Designed to accompany the Shear in Beams model)

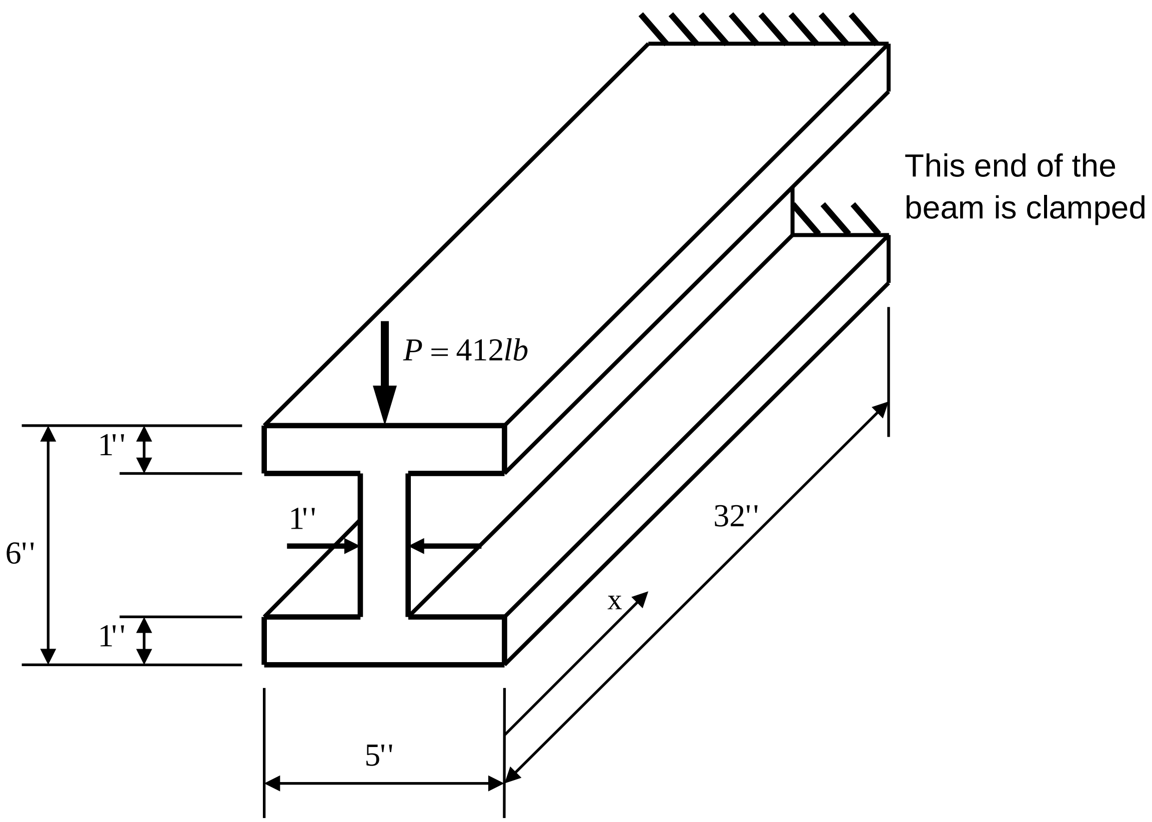
Problem Statement:

A cantilever beam, as shown below, is 32” long and it carries a downwards point load of 412 lb on its unsupported end. (The dimensions and force were chosen to produce simple calculations, and the units can be changed to centimetres and Newtons (N) or kN without changing the calculations.)

a) Determine the shear and moment in the beam as a function of position.

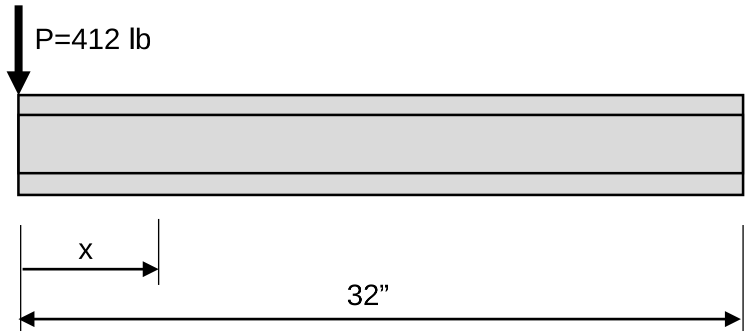
b) Calculate the bending stress at x=1” and x=2” (as measured from the end where the point load acts).

c) Calculate the shear flow acting on the beam cross section at x=1”.

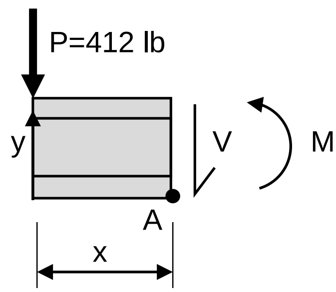


Solution:

a) Determine the shear and moment in the beam as a function of position.



Below, is a free-body diagram of the left segment of the beam, cut at position x.

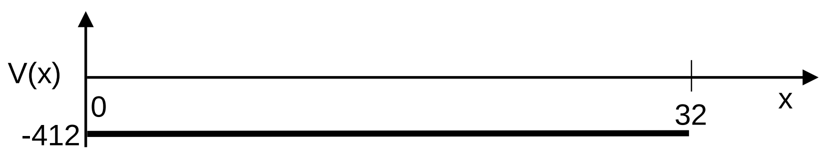


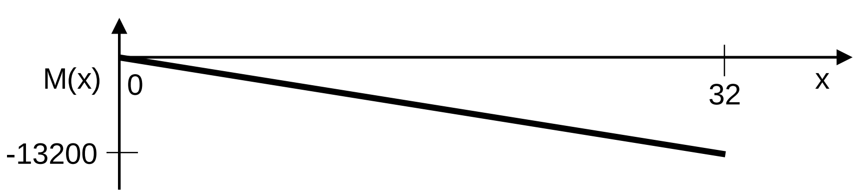
It allows us to calculate that

$$\sum F_y = 0 = -412 - V \Rightarrow V = -412lb$$

and

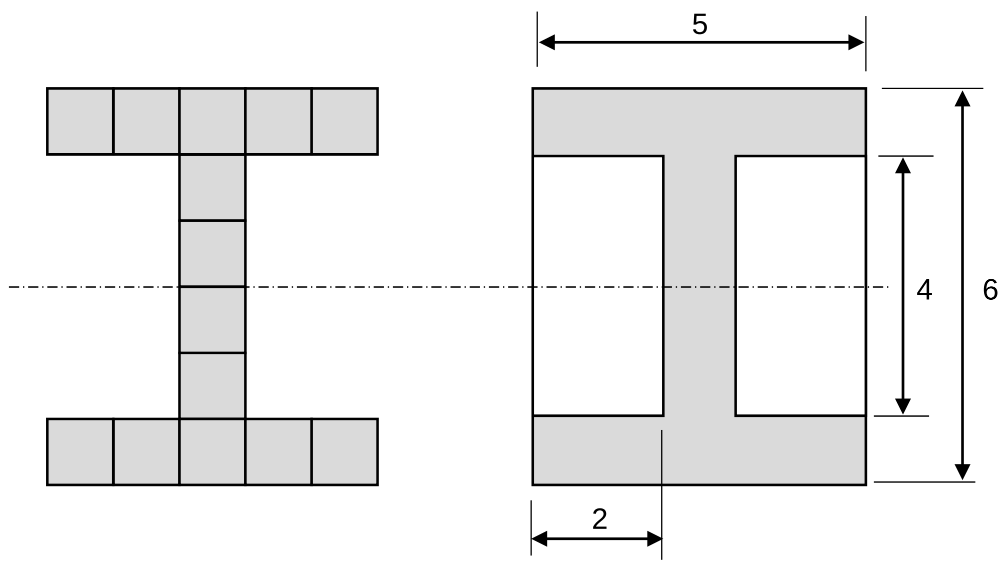
$$\sum M_A = 0 = M + 412\,x \Rightarrow M = -412x$$





b) Calculate the bending stress at x=1" and x=2" (as measured from the end where the point load acts).

For this beam, cross section,

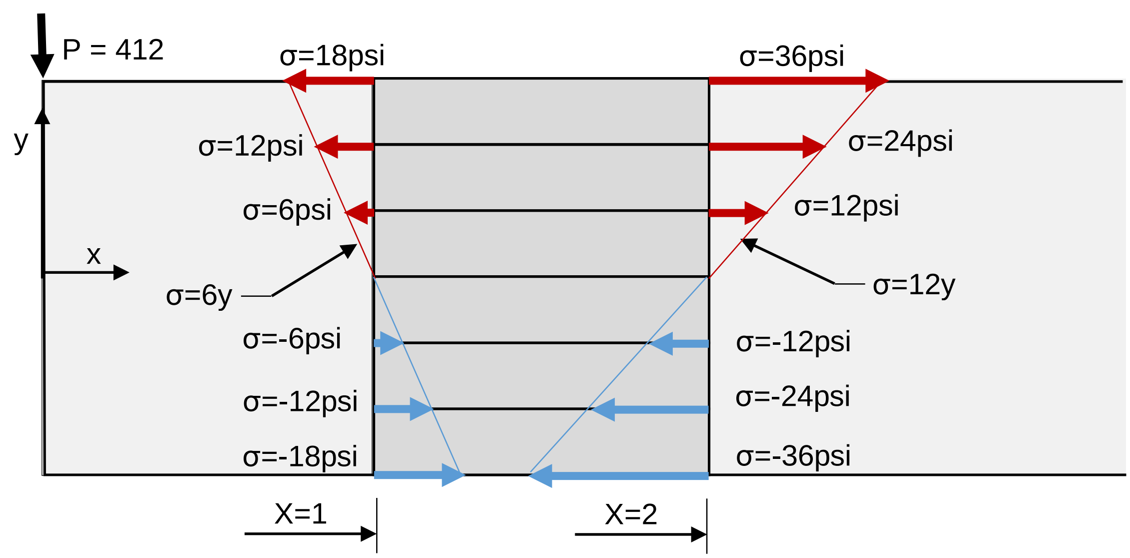


$$I = \frac{5 \cdot 6^3}{12} - 2\frac{2 \cdot 4^3}{12} = 68.67$$

The bending stress at position x is equal to

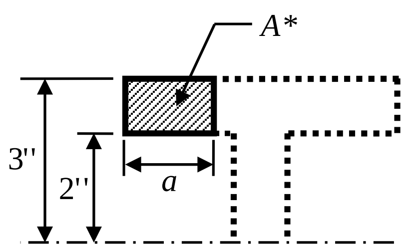
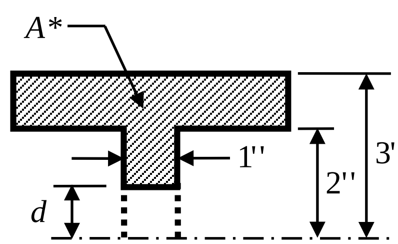
$$\sigma = \frac{My}{I} = \frac{(412x)\ (y)}{68.67} = 6xy$$

and the resulting stress distributions at x=1 and x=2 are shown in the figure below.



c) Calculate the shear flow acting on the beam cross section at x=1”.

The shear flow in a beam is given by the formula $q = \frac{VQ}{I}$

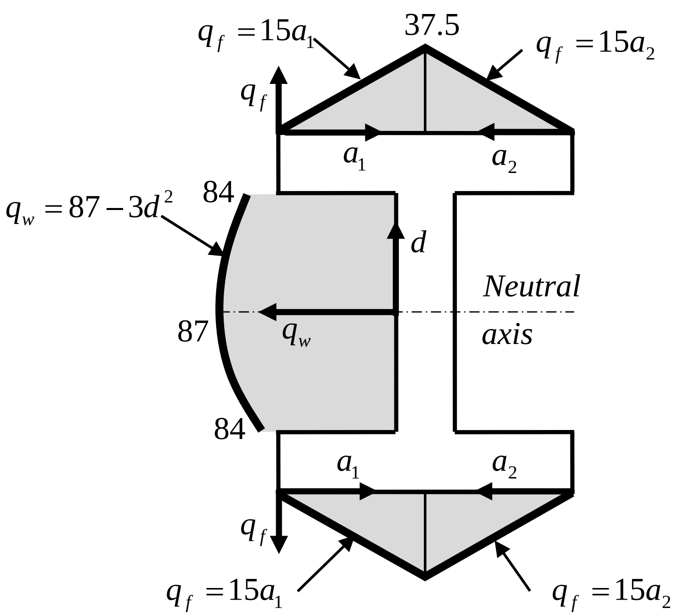
Analysis of the Flanges	**Analysis of the Web**
We cut through the thin dimension of the flange a distance “a” from the tip of the flange. The rectangular area that is shaded in the figure below becomes separated from the rest of the beam cross-section and is used to calculate Q_f.	We cut through the thin dimension of the web a distance “d” from the neutral axis of the beam cross-section, and the T-shaped area that is shaded in the figure below becomes separated from the rest of the cross-section and is used to calculate Q_w.
	
The Q value for the flanges is $Q_f = \int_{A*} y dA = \int_2^3 y(a dy)$ $= a\left\{\frac{y^2}{2}\right\}_2^3 = -\frac{5}{2}a$ and the shear flow (force per unit of cut length along the beam length) in the flanges is $q_f = \frac{VQ}{I} = \frac{412\left(\frac{5}{2}a\right)}{68.67} = 15a$ The shear stress (force per unit of cut area) in the flanges is $\tau_f = \frac{q_f}{t_f} = \frac{VQ}{It_f} = \frac{15a}{1} = 15a$ where t_f is the thickness of the flange.	The Q value for the flanges is $Q_w = \int_{A*} y dA = \int_2^3 y(5dy) + \int_d^2 y(1dy)$ $= \frac{25}{2} + 1\left\{\frac{y^2}{2}\right\}_d^2 = \frac{29}{2} - \frac{d^2}{2}$ and the shear flow (force per unit of cut length along the beam length) in the web is $q_w = \frac{VQ}{I} = \frac{412\left(\frac{29}{2} - \frac{d^2}{2}\right)}{68.67} = 87 - 3d^2$ The shear stress (force per unit of cut area) in the web is $\tau_w = \frac{q_w}{t_w} = \frac{VQ}{It_w} = \frac{= 87 - 3d^2}{1} = 87 - 3d^2$ where t_w is the thickness of the web.

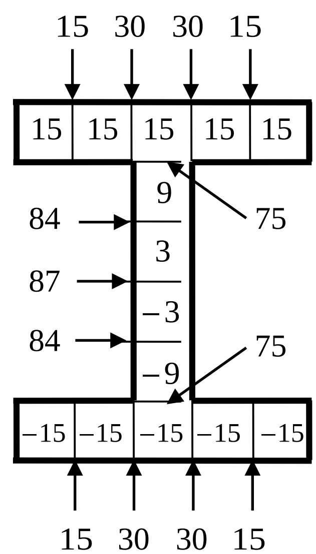
We can make a table of values showing the shear as a function of cutting plane position, "a".	We can make a table of values showing the shear as a function of cutting plane position, "d".

a	q_f	τ_f
0	0	0
1	15	15
2	30	30
2.5	37.5	37.5

d	q_w	τ_w
2	75	75
1	84	84
0	87	87

These values can then be plotted on a composite "shear flow diagram" which, by convention, is drawn as shown on the left below.

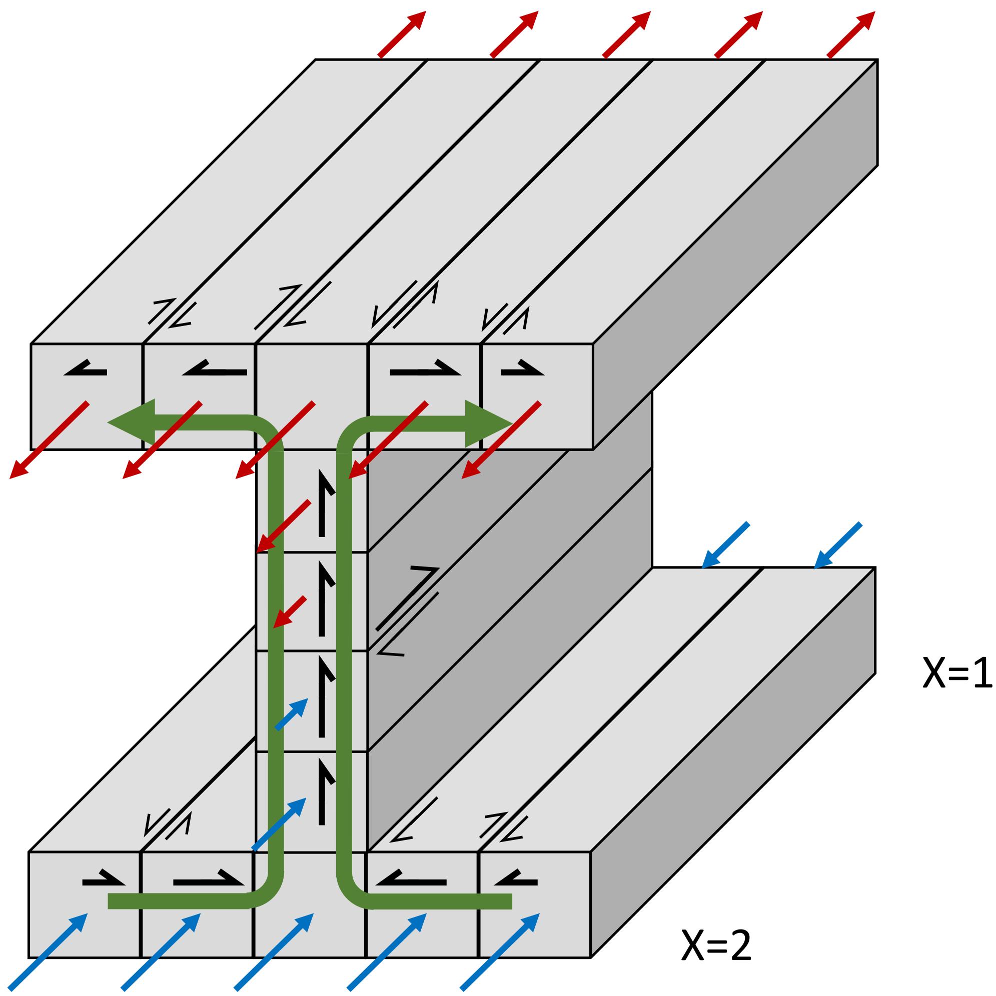




We can also consider the axial imbalance in each of 14 square blocks that one might consider to make up the beam cross-section – as in the model in the Shear in Beams video. In the figure on the right above, the axial imbalance in each block is shown in the square representing that block, and the total shear at each of the interfaces between blocks is labelled with a number and arrow in the figure below.

Notice how the discrete shear numbers in the figure on the right agree exactly with the shear values calculated at corresponding points by the formulas in the figure on the left. For example, the total shear at the root of the top right flange (a_2 =2) has a value of 30 in both cases. Also, the shear in the web 1" above the neutral axis is 84 in both cases.

To figure out the direction of the shear flow, consider the figure below.



The fiber at the top right of the beam experiences the bending stresses indicated by the red arrows. The tension on the back face of the fiber (x=1) is smaller than the tension on its front face (x=2), and so the rest of the beam must exert a shear force on that fiber in the backward direction, as indicated by the single-sided black shear arrow on the top of that fiber. At the same time, that fiber exerts a shear force on the second block from the top right corner that is in the opposite direction. Similar arguments can be made to determine the shear that acts elsewhere in the beam.

The shear on the top flange of the cross-section must act so that when that surface is viewed from above, the shear arrows at corners that are at 90° to each other go head to head and tail to tail. Thus the shear on the cross section must point outwards on the top flanges, as indicated by the black arrows. By similar arguments when the web is viewed from the side, we find that the shear on it is upwards, as shown by the black shear arrows on it. Finally, the shear on the bottom flanges flows inwards.

The curved green arrows show how the shear "Flows" over the cross-section.

The shear flow at the junction between the flanges and web is a bit more complex, and is not shown. It must be calculated using finite elements or some other advanced numerical method.

As a point of interest, if we integrate the vertical shear from the center of the lower flange (y=-2.5) to the center of the upper one (y=2.5), we obtain a total vertical shear force of

$$\int_{-2.5}^{2.5} q_w db = \int_{-3}^{3} \left\{87 - 36^2\right\} db = 435 - 31 = 404\, lb$$

which is in reasonable agreement with the expected value of 412 lb.

Notice also, that the shear flow is the same for any cross-section along the length of the beam since the shear function V(x) is constant with respect to x. The values of the bending stresses would vary with x, but the differences in the bending stresses between any one section and another one that is 1” further toward the clamped end of the beam would be identical those that arise in this analysis.