Automatic Traffic Shockwave Identification Using Vehicles’ Trajectories

by

Pedram Izadpanah*
PhD Candidate
Dept. of Civil and Environmental Engineering
University of Waterloo
Waterloo, ON Canada N2L 3G1
Phone : (519)888-4567, Ext. 36596
pizzadpan@uwaterloo.ca

Bruce Hellinga, PhD, PEng
Associate Professor
Dept. of Civil and Environmental Engineering
University of Waterloo
Waterloo, ON Canada N2L 3G1
bhellinga@uwaterloo.ca

Liping Fu PhD, PEng
Professor
Dept. of Civil and Environmental Engineering
University of Waterloo
Waterloo, ON Canada N2L 3G1
lfu@uwaterloo.ca

For presentation
88th Annual Meeting
Transportation Research Board
Washington, D.C.
January 2009

Compendium of papers of the 88th Annual TRB Conference held Jan. 11 – 15, 2009 in Washington, DC.

# Words: 5587 + 2000 (7 Figures and 1 Table) = 7587

*Corresponding author
ABSTRACT

Knowledge of the location and speed of shockwaves in a traffic stream provides insight into the formation and dissipation of congestion – information which is important for system managers. Furthermore, this information can be used to estimate and predict travel time for a section of a roadway. Most of the past efforts at identifying shockwaves have been focused on performing shockwave analysis based on fixed sensors such as loop detectors which are commonly used in many jurisdictions. However, latest advances in wireless communications have provided an opportunity to obtain vehicle trajectory data that potentially could be used to derive traffic conditions over a wide spatial area. This paper proposes a new methodology to detect and analyze shockwaves based on vehicle trajectory data. In the proposed methodology first the points that correspond to the intersection of shockwaves and trajectories of probe vehicles are identified and then a linear clustering algorithm is employed to group different shockwaves. Finally, a linear regression model is used to find propagation speed and spatial and temporal extent of each shockwave. The framework is evaluated using data obtained from a simulation of a signalized intersection and also real trajectory data from freeway US-101 near Los Angeles and shows promising results.

Keywords: traffic flow, shockwaves, freeway operation, vehicle trajectories, probe based traffic monitoring
Automatic Traffic Shockwave Identification Using Vehicles’ Trajectories

INTRODUCTION

The formation and dissipation of congestion are two phenomena that are important from traveler information system and congestion management perspectives. Predicting near future travel time requires the ability to identify congestion and to estimate the rate at which congestion is forming and/or dissipating. Furthermore, having knowledge of the formation and dissipation of congestion improves the ability to manage the system at work zone areas and other capacity bottlenecks and to respond more effectively to incidents.

Shockwave analysis is a technique to identify congested areas along a roadway and also to estimate the rate of the formation and dissipation of the congestion. A shockwave by definition is a boundary in a traffic stream that represents a discontinuity in the flow-density domain (6). Many researchers have focused on the application of shockwave analysis. However, most of their endeavors have been dedicated to shockwave analysis using data obtained from traditional fixed sensors such as loop detectors.

New developments within the wireless communication field provide the opportunity to obtain traffic condition information over a wide spatial area in near real time without the deployment of dedicated traffic sensors. Dedicated probe systems and mobile phone based traffic monitoring systems have already emerged in the market. Although these technologies are still in the evaluation or early deployment phase, they are being implemented in a number of jurisdictions and in some cases acceptable field results have been obtained (2). Moreover, research is being conducted on more advanced technologies such as Vehicle Infrastructure Integration (VII) or vehicular ad hoc networks (11). All of these technologies are able to provide positions of a sample of vehicles over time. The objective of this paper is to propose a methodology to detect “major” shockwaves in a traffic stream and provide speed and other attributes associated with them using vehicle trajectory data. The proposed methodology is technology independent and is able to deal with noisy data.

The rest of the paper is organized as follows. The next section deals with some background about origin of shockwaves, types of shockwaves, and relationship between trajectories of vehicles and shockwaves followed by the proposed methodology. The section on the proposed methodology consists of two sub sections: automatic shockwave detection and clustering. The paper concludes with the presentation of the results of applying the proposed methodology to a signalized intersection and a freeway section and conclusions.

BACKGROUND

One of the approaches to model traffic flow is to make an analogy with fluid flow. Lighthill, Whitham, and Richards were first to propose two similar models according to this analogy in two different seminal papers (5 and 10). Solutions to these models, which are based on the conservation equation of traffic flow and an assumed functional relationship between speed and density (or flow rate) will result in generation of shockwaves. By definition shockwaves are a boundary that shows discontinuity in the flow-density domain (6). The physical realization of a shockwave is points in time and space at which vehicles change their speed abruptly.

It can be shown that propagation speed of a shockwave that is created between two adjacent traffic states $A$ and $B$ can be calculated using the following equation:

$$\omega_{AB} = \frac{V_B - V_A}{D_B - D_A}$$  \hspace{1cm} (1)

where, $V_A$ and $V_B$ denote flow rate of traffic states $A$ and $B$ respectively and $D_A$ and $D_B$ represent density of traffic associated with traffic states $A$ and $B$ respectively.

Because the common source of traffic data in most jurisdictions is loop detectors, most researchers have focused on deployment of the continuum theory of traffic flow using data from loop detectors. Furthermore, the continuum theory of traffic flow was developed in the mid 1950s and received much attention in the literature; however, the theory was not widely deployed in practice (3). The main reason contributing to the lack of deployment of the continuum traffic flow models in practice is the difficulty associated with defining the initial boundary conditions. Moreover, these models require a closed network of sensors (e.g. loop detectors) in order to know the number of vehicles entering and exiting the network under consideration. However, even if sensors are installed in order to provide a closed network, sensor measurement errors and communication failures between the sensor and the central computer system will result in inaccurate solutions.
Emerging developments in the field of wireless communications provide the opportunity to obtain positions of a sample of vehicles over time. Using this information, trajectories of the probe vehicles can be constructed. If the points in time and space at which the speeds of the probe vehicles are changing abruptly were known, then the line resulting from connecting these points would represent the shockwaves in the traffic stream. This paper proposes a methodology by which the time and location of abrupt speed changes can be estimated, providing a method to detect different shockwaves in a traffic stream using data from probe vehicles.

METHODOLOGY

In the real world many shockwaves exist within a traffic stream. Most of these shockwaves are short-lived and/or insignificant from a traffic management or traveller information perspective. However, a number of these shockwaves not only last for a longer period of time but also represent the boundary between uncongested and congested flow states. Analyses of these shockwaves are important to estimate and predict traffic conditions accurately. These types of shockwaves are referred to as major shockwaves in this manuscript. In order to detect major shockwaves in a traffic stream a methodology is proposed in this section. The input data required for the proposed methodology are positions of a sample of vehicles over time. For each given time interval, the algorithm assesses whether or not the trajectory of a probe vehicle has intersected with a major shockwave and finds the coordinates of the point of the intersection in space and time. Having detected all of the shockwave intersection points in a particular time interval, a linear clustering algorithm is employed to group all the shockwave intersection points associated with each shockwave. A linear regression model is fitted to the data points within each cluster to estimate attributes (i.e. speed, positions and times of start and end) of such shockwave.

In this paper, it is assumed that (1) positions obtained from probe vehicles are accurate, (2) the lanes that probe vehicles are travelling on are known, and (3) a linear model can be used to model the propagation of shockwaves. The first assumption would probably affect the accuracy of the output of the proposed algorithm but the extent of the degradation can be quantified. Regarding the second assumption, congestion tends to be balanced in different lanes of a freeway (although this assumption may not be valid when congestion forms on the freeway as a result of insufficient capacity at an off-ramp). The third assumption is only true when vehicle headways in the traffic flow are uniform which is rarely true. However, it is anticipated that errors resulting from violating this assumption are relatively small.

According to the above description, the proposed methodology is divided into two parts: automatic shockwave detection and linear clustering. Details of each are provided in the following sections.

Automatic Shockwave Detection

An iterative two-phase piecewise or switching regression is used to detect shockwaves. This type of regression is used when two lines with different slopes fit the data and the “joint point” or the “change point” is not known a priori. In other words, suppose \( n \) pairs of data points \( (t_i, x_i), i = 1, \ldots, n \) are available, where \( t_i \) is the time at which the probe vehicle is reported at position \( x_i \) along the roadway. Furthermore, it can be assumed that \( t_i \) are ordered in a way that \( t_1 \leq t_2 \leq \cdots \leq t_n \). Then, \( t \) and \( x \) can be related to each other according to the following set of equations:

\[
\begin{align*}
x_1 &= a_1 + b_1 t_i, \quad t_i \leq t_0 \\
x_2 &= a_2 + b_2 t_i, \quad t_i > t_0
\end{align*}
\]

Where \( (t_0, x_0) \) is the unknown joint point (See trajectory of vehicle B in FIGURE 2). This problem has many applications in different fields of science such as biology and econometrics (17 and 18). Quandt (8) was the first who addressed this problem. In the context of probe vehicles, the joint point is the point at which the speed of the vehicle has changed and as such likely represents the shockwave boundary between two flow states. These points are referred to as “inflection points”. In this research, the two phase piecewise linear regression model proposed by Vieth (17) is applied to the time series of probe vehicle position data in order to find inflection points on the trajectories of these vehicles.

Assume that a sample of \( P \) probe vehicles is available during time period \( \Delta \). For each probe vehicle \( p \) a set of points, \( \Gamma^p = \{(t_i, x_i) | i = 1, \ldots, n^p\} \), in time and space is available during time interval \( \Delta \). Time interval \( \Delta \) can also be discretized into a number of smaller time steps. The mathematical program to fit a two-phase linear regression to the points in set \( \Lambda^p \) can be formulated as follows where \( \Lambda^p \) is a subset of \( \Gamma^p \) and includes data points associated with one or more time steps depending on the...
number of the inflection points detected in previous time steps for probe $p$. Set $A^p$ is defined formally later on with respect to FIGURE 1.

$$\text{Min } RRS = \sum_{t_i \leq t_0} [x_i - (a_1 + b_1 t_i)]^2 + \sum_{t_j < t_0} [x_i - (a_2 + b_2 t_j)]^2$$  \hspace{1cm} (4)$$

s.t.

1) $(t_j, x_j) \in A^p \quad \forall i = 1, \ldots, \|A^p\|$  
2) $t_j \leq t_0 < t_{j+1} \quad j = 1, \ldots, \|A^p\| - 3$

3) $\theta \times [x_0^j - (a_1 + b_1 t_0)] = 0$

4) $\theta = \begin{cases} 
0 & \text{if no inflection point has been found so far}, \\
1 & \text{if at least one inflection point has been found}
\end{cases}$

where $\|A^p\|$ represents dimension of set $A^p$ and $(t_0^j, x_0^j)$ is the last inflection point that has been found in previous time steps.

In the above mathematical program, the objective function is a minimization of the total residual sum of squares associated with both regression lines fitted to the data. Furthermore, $t_0$ is the time component of the inflection point. The first constraint guarantees that the lines are fitted to the data in set $A^p$ and the second constraint ensures that the inflection point exists within the time limits of the data in $A^p$. In other words, the above program finds the best piecewise linear regression, for which the inflection point falls within the available data. The third and fourth constraints guarantee physical continuity in the trajectory of the probe vehicle. In other words, when an inflection point is detected, the next regression line in the next time step must pass through this point to maintain continuity.

In order to solve the above program, the following iterative algorithm can be used:

Step 0: Set $j = 3$ and choose set $A^p$.

Step 1: while $j \leq \|A^p\| - 3$

- Partition $A^p$ into two mutually exclusive and collectively exhaustive sets: $\lambda_1 = \{(t_i, x_i) | i = 1, \ldots, j\}$ and $\lambda_2 = \{(t_i, x_i) | i = j + 1, \ldots, \|A^p\|\}$.
- Perform two regular linear regressions in order to find the best lines fitted to $\lambda_1$ and $\lambda_2$. If $\theta$ is 1, then the piecewise regression line has to be constrained to pass through the inflection point found in the previous time period.
- Calculate the objective function of mathematical program(4), $RSS$.
- Set $j := j + 1$

Step 2: Choose a piecewise linear regression with the smallest $RSS$.

$A^p$ used above is defined in FIGURE 1 which illustrates all the components of the proposed algorithm to automatically detect shockwaves.
Assume that in FIGURE 1 time interval $\Delta$ can be divided into $K$ time steps. Set $p_k\Gamma$ in this figure includes all position measurements associated with probe vehicle $p$ that are obtained during time step $k$. In this flow chart, $p\Pi$ is the set containing all inflection points for probe vehicle $p$. In other words, this set is the output of the above algorithm for probe vehicle $p$ over time interval $\Delta$. Set $p\Lambda$ includes data from the most recently found inflection point to the end of data points associated with the current time step. FIGURE 2 shows two cases to clarify the definition of set $p\Lambda$. This figure shows trajectories of probe vehicles $A$ and $B$ during time interval $\Delta$. The time interval is also divided into 4 time steps. Probe vehicle $A$ has been traveling at approximately a constant speed. Consequently, the algorithm does not find any inflection point and at time step $k = 4$ set $A^A$ includes all available data points ($A^A = \{(t_i,x_i) | i = 1,\cdots,9\}$). In the case of probe vehicle $B$ an inflection point was found in time step $k = 2$ and after that the probe vehicle travelled at approximately a constant speed resulting in no other inflection point until $k = 4$. Therefore, set $A^B$ contains all data point from the most recently found inflection point to data point associated with the current time interval inclusive ($A^B = \{(t_0,x_0) \cup \{(t_i,x_i) | i = 7,\cdots,15\}\}$).

**FIGURE 1 Proposed algorithm to estimate inflection points.**

In this flow chart, $p\Pi$ is the set containing all inflection points for probe vehicle $p$. In other words, this set is the output of the above algorithm for probe vehicle $p$ over time interval $\Delta$. Set $A^P$ includes data from the most recently found inflection point to the end of data points associated with the current time step. FIGURE 2 shows two cases to clarify the definition of set $A^P$. This figure shows trajectories of probe vehicles $A$ and $B$ during time interval $\Delta$. The time interval is also divided into 4 time steps. Probe vehicle $A$ has been traveling at approximately a constant speed. Consequently, the algorithm does not find any inflection point and at time step $k = 4$ set $A^A$ includes all available data points ($A^A = \{(t_i,x_i) | i = 1,\cdots,9\}$). In the case of probe vehicle $B$ an inflection point was found in time step $k = 2$ and after that the probe vehicle travelled at approximately a constant speed resulting in no other inflection point until $k = 4$. Therefore, set $A^B$ contains all data point from the most recently found inflection point to data point associated with the current time interval inclusive ($A^B = \{(t_0,x_0) \cup \{(t_i,x_i) | i = 7,\cdots,15\}\}$).
The presence or absence of an inflection point is determined statistically. In FIGURE 3 the piecewise linear regression model defined by equation (4) has been applied to both the trajectory data from two probe vehicles. However, in the case of probe vehicle $A$ the piecewise linear regression is not statistically different from a single regime linear regression. Consequently, the one regime linear regression is chosen as the better model describing spatial and temporal movement of this probe vehicle and no point of inflection is defined. On the other hand, for probe vehicle $B$ a piecewise linear regression model is statistically superior to the single regime model.

**FIGURE 2 Two examples to show set $A^P$.**

The presence or absence of an inflection point is determined statistically. In FIGURE 3 the piecewise linear regression model defined by equation (4) has been applied to both the trajectory data from two probe vehicles. However, in the case of probe vehicle $A$ the piecewise linear regression is not statistically different from a single regime linear regression. Consequently, the one regime linear regression is chosen as the better model describing spatial and temporal movement of this probe vehicle and no point of inflection is defined. On the other hand, for probe vehicle $B$ a piecewise linear regression model is statistically superior to the single regime model.

**FIGURE 3 Statistically significant piecewise linear regression.**
The following hypothesis test can be applied to check whether or not a piecewise linear regression is statistically different from a single regime model (17):

\[
H_0: a_1 = a_2 \text{ and } b_1 = b_2 \\
H_1: a_1 \neq a_2 \text{ or } b_1 \neq b_2
\]

The following statistic can be used to perform the above hypothesis test:

\[
F = \frac{(RSSL - RSS)/3}{RSS/(\|A^p\| - 4)}
\]

Where, \( RSSL \) in equation (6) is the residual sum of squares of the single regime linear regression fitted to all point in \( A^p \) and \( RSS \) is the residual sum of squares associated with the piecewise linear regression which is defined by the objective function of equation (4). The F-statistic defined by equation (6) can be compared with an F-test table value with 3 and \( \|A^p\| - 4 \) degrees of freedom at a given level of confidence (e.g., 95%). If the F-statistic obtained using equation (6) is larger than the table value, the null hypothesis is rejected implying that the piecewise linear regression is statistically different from the single regime linear regression and a point of inflection has not been found.

**Linear Clustering**

At this stage, all inflection points associated with all probe vehicles in set \( P \) have been found. However, there are still two main challenges that need to be addressed. First, not every inflection point is part of a major shockwave. Second, the points may be associated with multiple shockwaves. A data filtering procedure is proposed to address the first challenge. A linear clustering algorithm is proposed to address the second challenge.

**Data Filtering**

The main objective of the data filtering procedure is first to identify inflection points that are more likely to be part of a “major” shockwave and second to group the resulting points into two distinctive groups to facilitate the linear clustering algorithm which is described in the next section.

To identify the inflection points that reflect major shockwaves, the difference in average vehicle speed before and after the inflection point (i.e., slopes of the two regression lines) can be computed. If the absolute magnitude of this change in speed is greater than a specified threshold, then the point of inflection is assumed to be associated with a major shockwave and is classified as a shockwave intersection point.

The shockwave intersection points can be divided into two broad groups to facilitate and expedite the clustering procedure. Assume that \( S^u_i \) and \( S^d_i \) denote speed of a probe vehicle upstream and downstream of shockwave intersection point \( i \) respectively. Shockwave group 1 includes the shockwave intersection points with \( S^u_i > S^d_i \) and conversely shockwave group 2 contains the shockwave points with \( S^u_i < S^d_i \). The clustering algorithm, described in the next section, is applied to the two groups separately to improve computational efficiency and clustering accuracy.

**Clustering Algorithm**

The data filtering procedure has reduced the inflection points to the shockwave intersection points. Furthermore, the data has been categorized into two groups. The resulting shockwave intersection points need to be grouped into different shockwaves in order to find the characteristics of each shockwave.

Earlier in the background section, it was discussed that under certain assumptions linear models can be used to model the propagation of shockwaves in a traffic stream. However, most clustering algorithms such as \( k \)-means algorithm (1) identify sparse and crowded datasets and are not appropriate for datasets with linear patterns (12). Consequently, a linear clustering algorithm is adopted to cluster shockwave intersection data points to identify shockwaves. A number of researchers have proposed different algorithms to cluster datasets with linear patterns (12, 7, and 10). In this paper a modified version of the methodology proposed by Spath (10) is used to cluster shockwave intersection points.

Assume that \( \Omega_l = \{(t_i, x_i) | i = 1, \cdots, m_l \} \) is a set which contains all shockwave intersection points corresponding to shockwave group \( l \) where \( l \) is either 1 or 2. The linear clustering problem can be formulated as follows:
\[
\text{Min} \sum_{j=1}^{n} \sum_{i \in C_j} \left[ x_i - (\alpha_{C_j} + \beta_{C_j} t_i) \right]^2
\]  
(7)

s.t.
1) \( C_j \subseteq \Omega_j \) \quad \forall j = 1, \ldots, n
2) \( \|C_j\| \geq 3 \) \quad \forall j = 1, \ldots, n
3) \( C_j \cap C_k = \emptyset \) \quad \forall j, k = 1, \ldots, n and \( j \neq k \)
4) \( \bigcup_{j=1}^{n} C_j = \Omega_l \)

where, \( n \) denotes the number of clusters to be identified and \( C_j \) represents a cluster.

The objective function of the above mathematical problem is a minimization of the total sum of the squared error of all clusters provided that \( x = \alpha_{C_j} + \beta_{C_j} t_i \) is the regression line fitted to all data points in cluster \( C_j \). Constraints 1 and 2 guarantee that data point of clusters are a subset of \( \Omega \) and at least 3 data points exist in each cluster respectively. Constraints 3 and 4 ensure that the clusters are mutually exclusive and collectively exhaustive.

The mathematical problem (7) is a nonlinear integer program which is intrinsically difficult to solve. However, this problem can be solved using the following algorithm proposed by Spath (10):

Step 1: Choose an initial feasible solution.
Step 2: For each cluster \( C_j \), perform a regression analysis to determine \( \{a_{C_j}, b_{C_j}\} \)
Step 3: For a randomly selected \( i \in C_j \) examine if there are clusters \( C_p \) with \( p \neq j \) such that shifting \( i \) from \( C_j \) to \( C_p \) reduces the objective function. If so, choose \( C_r \) that maximizes the reduction. Then, set \( C_j = C_j - \{i\}, C_p = C_p \cup \{i\} \). Continue this step until all data points are visited.
Step 4: Repeat Step 2 for a given number of times or until no reduction in objective function is achieved.

Set \( \{\beta_{C_j} \mid j = 1, \ldots, n\} \) contains the estimated speed of the shockwaves associated with each cluster.

It should be noted that this clustering algorithm has two limitations: first, the performance of the linear clustering algorithm highly depends on the initial feasible solution. One method that can be used to create a “good” initial solution is to recognize the fact that shockwave points which are close together spatially and are far from each other temporally cannot be part of the same shockwave.

Second, the number of clusters is assumed to be known. However, in reality this is not the case. One approach to addressing this limitation is to apply the clustering algorithm for different values of \( n \) and then select the solution that maximizes the total marginal benefit. In other words, the optimal number of clusters, \( n \), is selected according to the fact that increasing the number of clusters from \( n-1 \) to \( n \) results in a maximum reduction in the objective function defined in Equation (7).

**NUMERICAL RESULTS**

The proposed framework was applied to detect shockwave in two case studies namely, simulation of a signalized intersection and real data from freeway US-101 near Los Angeles.

**Signalized Intersection**

A single exclusive through lane of a signalized intersection was simulated using the INTEGRATION simulation model (13 and 14). The signal was controlled by a fixed time two phase signal timing plan with a cycle length of 100 seconds; an effective green of 45 and 51 seconds, total lost time of 4 seconds and offset of 52 seconds were chosen. Saturation flow rate was 1900 vehicles per hour per lane. Traffic demand was assumed to be constant at 700 vph and consisted of only passenger cars. Furthermore, vehicles were generated with uniform headways. The approach link was 1 km in length and had a free flow speed of 60 km/h; a speed at capacity of 40 km/h and jam density of 125 veh/km/lane. Furthermore, the approach is controlled by phase 1 of the signal.

The INTEGRATION simulation model uses a single regime four parameter macroscopic traffic flow model proposed by Van Aerde (15 and 16). Van Aerde’s model is defined by the following set of equations:
\[ D = \frac{1}{C_1 + \frac{C_2}{S_f - S} + C_3 S} \] (8)

\[ C_1 = k C_2 \] (9)

\[ C_2 = \frac{1}{D_f \left( k + \frac{1}{S_f} \right)} \] (10)

\[ k = \frac{2 S_c - S_f}{(S_f - S_c)^2} \] (11)

\[ C_3 = \frac{1}{S_c} \left( -C_1 + \frac{S_c}{V_c} - \frac{C_2}{S_f - S_c} \right) \] (12)

where,
- \( D \) = Density (veh/h/lane)
- \( S \) = Speed (km/h)
- \( S_f \) = Free flow speed (km/h)
- \( S_c \) = Speed at capacity (km/h)
- \( V_c \) = Capacity volume or saturation flow rate (vph)
- \( D_j \) = Jam Density (veh/km/lane).

Using Equations (8) through (12), the fundamental equation of traffic, and equation for calculating propagation speed of shockwaves, Equation (1), a shockwave diagram for this intersection approach can be constructed for a typical signal cycle (FIGURE 4).

In FIGURE 4, \( A \), \( B \), and \( C \) represent traffic states that exist upstream of the traffic signal. \( \omega_{AB} \), \( \omega_{BC} \), and \( \omega_{CA} \) denote speed of propagation of shockwaves that are created between traffic states \((A, B)\), \((B, C)\) and \((C, A)\) respectively.

The proposed algorithm was applied to the simulation output. All vehicles were treated as probe vehicles. The time intervals were chosen to be 5 minutes and each time step within the time intervals was 5 seconds. A 95\% level of confidence was used to perform the hypothesis test explained by Equations (5) and (6). Furthermore, the thresholds to identify shockwave intersection points among the inflection points were chosen to be 18 (km/h) and 14.4 (km/h) for shockwave group 1 (backward forming shockwave in this case) and shockwave group 2 (backward recovery shockwave in this case study) respectively. Because vehicles do not change their speed promptly when they meet a

**FIGURE 4 Shockwave diagram for one cycle of the simulated approach.**

In FIGURE 4, \( A \), \( B \), and \( C \) represent traffic states that exist upstream of the traffic signal. \( \omega_{AB} \), \( \omega_{BC} \), and \( \omega_{CA} \) denote speed of propagation of shockwaves that are created between traffic states \((A, B)\), \((B, C)\) and \((C, A)\) respectively. The proposed algorithm was applied to the simulation output. All vehicles were treated as probe vehicles. The time intervals were chosen to be 5 minutes and each time step within the time intervals was 5 seconds. A 95\% level of confidence was used to perform the hypothesis test explained by Equations (5) and (6). Furthermore, the thresholds to identify shockwave intersection points among the inflection points were chosen to be 18 (km/h) and 14.4 (km/h) for shockwave group 1 (backward forming shockwave in this case) and shockwave group 2 (backward recovery shockwave in this case study) respectively. Because vehicles do not change their speed promptly when they meet a
shockwave, more than one shockwave intersection point is expected to be obtained according to the above criteria. Consequently, for shockwave group 1 for each probe vehicle the last shockwave intersection point is considered for clustering and for shockwave group 2 for each probe vehicle the first shockwave intersection point is taken into account.

FIGURE 5 shows 5 minutes of simulation data. Randomly selected colors have been used for better illustration of individual vehicles’ trajectories.

The red circles in this figure represent shockwave intersection points associated with backward forming shockwaves and the green crosses denote shockwave intersection points corresponding to recovery shockwaves. The lines passing through each group of inflection points, representing the identified shockwaves, are the result of the clustering module. The propagation speed of each shockwave is also shown on the figure. Furthermore, the algorithm is also able to provide other attributes of the shockwaves such as time and location of the start and the end in addition to average speed of vehicles upstream and downstream of the shockwave during current time interval.

As can be seen in FIGURE 5, the framework could successfully identify both backward forming and backward recovery shockwaves but failed to identify the forward moving shockwaves. The main reasons for this might be (1) the propagation speed of the forward moving shockwave is higher than the other shockwaves; (2) the shockwave does not represent a boundary between two congested traffic states; and (3) this type of shockwave lasts for a shorter period of time.

A direct comparison of the shockwave speeds estimated by the proposed method and the true shockwave speeds is not possible as the INTEGRATION model does not explicitly output shockwave speeds and there does not exist a formal method of computing shockwave speeds on the basis of vehicle trajectory data. However, a comparison with the analytical estimates is possible (TABLE 1). As can be seen in this table the proposed framework was able to estimate the speed of the backward forming shockwaves accurately (<1 % difference). However, the relative difference between the analytical estimate and the results obtained from applying the framework is 27.3% in case of the recovery shockwave. It is expected that using different thresholds to differentiate between the inflection points and shockwave intersection points would improve the similarity between the proposed framework and the analytical estimates, however, the analytical estimates are themselves an approximation and subject to errors.

FIGURE 6 shows the shockwave diagram obtained from analytical estimates and applying the proposed framework. This figure can be used to temporally and spatially compare the analytical estimates and output of the proposed framework for one cycle of the traffic signal. In this figure the solid black and red lines represent the shockwave diagram based on the analytical framework and the
proposed framework respectively. Furthermore, in this figure the red circles represent the beginning and end of the shockwave intersection points identified for both backward forming and recovery shockwaves. As can be seen the main difference is the fact that the proposed framework did not capture the forward moving shockwave.

TABLE 1 Comparison of Analytical Estimates of Shockwave Speed With the Results Obtained From Applying the Proposed Methodology

<table>
<thead>
<tr>
<th>Shockwave Type</th>
<th>Analytical Estimate of Propagation Speed (Km/h)</th>
<th>Average Speed Obtained by the Proposed Framework (Km/h)</th>
<th>Relative Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Backward Forming</td>
<td>-6.20</td>
<td>-6.18</td>
<td>0.32</td>
</tr>
<tr>
<td>Recovery</td>
<td>-24.52</td>
<td>-17.83</td>
<td>27.3</td>
</tr>
</tbody>
</table>

To further evaluate the performance of the proposed framework, real trajectory data available from the NGSIM dataset (5) for one lane of freeway US-101 was used. The polling interval between two consecutive locations in the NGSIM trajectory dataset is 0.1 second. The time intervals were chosen to be 5 minutes and each time step within the time intervals was 10 seconds. A 95% level of confidence was used to perform the hypothesis test explained by Equations (5) and (6). Furthermore, the thresholds to identify shockwave intersection points among the inflection points were chosen to be 16.5 (Km/h) and 20.8 (Km/h) for shockwave group 1 (backward forming shockwave in this case) and shockwave group 2 (backward recovery shockwave in this case study) respectively.

FIGURE 7 shows trajectories of probe vehicles which were depicted using randomly assigned colors. The red circles represent inflection points corresponding to backward forming shockwaves and green crosses demark inflection points associated with recovery shockwaves.

Freeway

To further evaluate the performance of the proposed framework, real trajectory data available from the NGSIM dataset (5) for one lane of freeway US-101 was used. The polling interval between two consecutive locations in the NGSIM trajectory dataset is 0.1 second. The time intervals were chosen to be 5 minutes and each time step within the time intervals was 10 seconds. A 95% level of confidence was used to perform the hypothesis test explained by Equations (5) and (6). Furthermore, the thresholds to identify shockwave intersection points among the inflection points were chosen to be 16.5 (Km/h) and 20.8 (Km/h) for shockwave group 1 (backward forming shockwave in this case) and shockwave group 2 (backward recovery shockwave in this case study) respectively.

FIGURE 7 shows trajectories of probe vehicles which were depicted using randomly assigned colors. The red circles represent inflection points corresponding to backward forming shockwaves and green crosses demark inflection points associated with recovery shockwaves.

The propagation speed of each shockwave found by applying the proposed methodology is shown on FIGURE 7. Lu and Skabardonis (5) proposed an algorithm to estimate the propagation speed of shockwaves on freeways based on vehicle trajectory data. They also tested the algorithm using the NGSIM dataset for the same lane and section of freeway. Lu and Skabardonis used a filtering technique to smooth speed-time trajectories of vehicles in order to find the minima associated with each trajectory. Then, a clustering algorithm is employed to find speed corresponding to each shockwave. It should be noted that these researchers found only the backward forming shockwaves. Lu
and Skabardonis found that the backward forming shockwaves are parallel to each other with a propagation speed of 18.3 Km/h. The average propagation speed of the backward forming shockwaves as determined from the proposed framework is 13.8 ft/Sec or 15.1 Km/h. The relative difference between the proposed framework and the estimated obtained by Lu and Skabardonis is 17.5%.

FIGURE 7 Trajectory of probe vehicles, shockwave intersection points and detected shockwaves on one lane of freeway US-101 (NGSIM data).

CONCLUSIONS AND FUTURE RESEARCH

This paper proposes a framework to detect and analyze shockwaves in a traffic stream. The framework consists of two main parts: automatic shockwave detection and linear clustering. The automatic shockwave detection part uses a piecewise linear algorithm in order to find the points in time and space that a trajectory of a vehicle intersects with a major shockwave. The linear clustering recognizes that points may be associated with different shockwaves and bounds them together in order to estimate attributes of the shockwaves such as speed, the time and location that they started and ended.

The framework was tested with data obtained from the simulation of a signalized intersection and the results were compared with analytical estimates of shockwave propagation speeds. In the case of the backward forming shockwaves, the estimates based on the proposed framework and analytical estimates were 0.32% different and in the case of the recovery shockwaves, they were 27.3% different. The proposed framework did not identify the forward moving shockwave. The proposed framework was also applied to real trajectory data associated with freeway US-101 near Los Angeles which was extracted from the NGSIM dataset. Estimates of shockwave speeds are within 17.5% of those estimated by Lu and Skabardonis for the same dataset.

On the basis of the performance results presented in this paper, it is anticipated that the main application of the proposed algorithm is to suburban and rural freeways that are not monitored by traditional sensors such as loop detectors. Using emerging technologies such as mobile phone based traffic monitoring systems and the proposed algorithm the formation and dissipation of congestion can be monitored for such freeways. Furthermore, it is expected that the ability to accurately identify shockwave locations and speeds will enable the accurate estimation and prediction of freeway section travel times.

A number of improvements to the current algorithm are quite conceivable in the future. The outlier shockwave intersection points (or inflection points) that are mistakenly found or are isolated have to be filtered out. The isolated shockwave intersection points could result from local disturbances in the traffic stream that are somewhat insignificant from the perspective of travel time estimation and prediction.

As future research, the proposed methodology should be used to estimate and predict travel time on freeways. The sensitivity of travel time prediction and estimation to a few parameters should be performed. These parameters include (1) accuracy of locations reported; (2) number of probe vehicles.
required; (3) polling interval between two consecutive reported locations of a probe vehicle; (4) effect of considering probe vehicles traveling on different lanes together; (5) time interval and subinterval in the proposed methodology, and (6) different threshold values to identify shockwave intersection points among the inflection points.

References