Day-to-Day Variability of Signalized Intersection Performance during the Peak Hour

By
Zeeshan Abdy and Bruce Hellinga

Proceedings of the ITE Canadian District Annual Conference held May 6-10, 2007 in Toronto, Canada.

ABSTRACT:

Traffic Signal timing plans are typically developed on the basis of volume counts on a single day. These volume counts are used to determine the peak hour factor (PHF) and the peak hour turning movement volumes, which are then typically used as input to analysis methods defined in design manuals such as the Canadian Capacity Guide (CCG) and Highway Capacity Manual (HCM). Both CCG and HCM methodology are deterministic in that they ignore potentially important variations including day-to-day variation in peak hour volume. The lack of consideration of this variability may be because (a) it is assumed that the impact of the variability is small; and/or (b) methods have not been developed by which the variability can be considered.

This paper presents finding of a study that has been conducted to quantify the existence and impact of day-to-day variability of intersection peak hour approach volumes. Distributions were statistically fit to the variability observed in the field data and then used as inputs to a Monte Carlo Simulation (MCS) of a hypothetical signalized intersection. The results show that the variation in the intersection performance (i.e. delay) increases dramatically as the Degree of Saturation (X) changes from 0.6 to 1.0. In addition, the variation in intersection delay increases with increasing correlation of the approach volumes. This impact is increasingly pronounced as the intersection design quality of service decreases (i.e. moves to X = 1.0).

The large variation in intersection performance, particularly as approach volumes increase, implies that reliably estimating intersection delays may require turning movement counts from a large number of days. This may be particularly important when using estimates of intersection performance to decide on the necessity of intersection improvement and/or the assessment of developer fees.

Results from all Degree of Saturation scenarios indicated that computing the intersection delay based on the average volumes, and ignoring the variability of these volumes, underestimates the true average intersection delay by as much as 18%. Furthermore, the estimation error is largest when the intersection degree of saturation is close to 1.0.
1. INTRODUCTION

Both the Canadian Capacity Guide (CCG) and the Highway Capacity Manual (HCM) use average vehicle delay (seconds per vehicle) for defining signalized intersection performance. This information is then used to define intersection performance in terms of six levels of service.

Intersection performance (i.e. delay) is a function of many factors including: signal timing plan, turning movement traffic demands, traffic stream composition, pedestrian volumes, intersection geometry, temporal variation in traffic demands, the headway distribution of each traffic stream, driver characteristics, weather and road surface conditions and visibility. Some of these factors are invariant for a given intersection operating under a defined signal control strategy (e.g. geometry and signal timing plan) while others vary (e.g. weather, traffic demands, etc.).

Some of this variability is captured (or controlled for) by the intersection analysis methodology. For example, traffic demands vary by time of day, but this is controlled for by applying the analysis method for the peak hour volume and utilizing the peak hour factor (PHF). Weather conditions are controlled for by assuming ideal weather conditions. The random variability of vehicle arrivals (i.e. headway distribution of the approach traffic streams) are assumed to be Poisson and then the influence of nearby upstream signalized intersections in terms of creating platoons is considered.

However, variability of other factors is not considered including the day-to-day variability in the traffic volumes, PHF, and saturation flow rate. This raises a number of issues about appropriate criteria for intersection control evaluation and design including:

1. Traffic engineers typically design signal timing plans to achieve a prescribed level of service (say LOS C). This is interpreted to mean that the timing is designed to provide, on average, LOS C. Signal designs are determined on the basis of turning movement volumes that are assumed to reflect average peak hour demands. But, does selecting a signal timing plan to provide LOS C on the basis of average turning movement volumes, provide an average intersection performance of LOS C?

2. What is the distribution of the performance provided by the signalized intersection? For example, how frequently will the intersection experience LOS A, B, C, D, E or F during the weekday peak hour?

3. In practice, traffic engineers typically collect turning movement volume count data in 15 minute intervals over a peak period. On the basis of these data, the peak hour is identified, the peak hour volumes are extracted, and the peak hour factor is computed. If the peak hour of each weekday is considered as a single outcome (or observation) then signal analysis and design is generally conducted on the basis of turning movement counts that represent a single observation from a distribution. If a desired level of accuracy is desired in terms of estimating the average intersection delay, over how many days should turning movement counts be obtained?
In this paper we answer these questions using empirical data to quantify the distribution of day-to-day peak hour traffic volumes and the degree of statistical correlation between approach volumes. Then these data are used as input to a Monte Carlo simulation to determine the associated distribution of intersection delay.

The next section provides the background of the HCM delay estimation expressions, previous work examining the sensitivity of intersection performance to variability of key input parameters, and the study methodology. Section 3 provides a description of the data used in the study and the characterization of the empirical distributions. Section 4 provides the results of the Monte Carlo simulation and Section 5 provides conclusions and recommendations.

2. ANALYSIS METHODOLOGY

2.1 Background

Methods to analyze the performance of a given signal timing plan, and to develop optimal plans, have been developed since the 1950s and are now embedded in design manuals such as the Highway Capacity Manual (HCM) and Canadian Capacity guide (CCG).

The methods in the HCM and CCG are based on the work of Webster (1958) who first developed a relationship between signal timings, and traffic characteristics and intersection performance (i.e. delay). Webster’s original expression for delay is given as

\[
d' = d_i + \frac{x^2}{2\lambda(1-x)} - 0.65\left(\frac{C}{\lambda^2}\right)^{1/3}X^{2+5g/C}
\]

(1)

\[
d_i = \frac{0.5C\left(1 - \frac{g}{C}\right)^2}{1 - X_i \frac{g}{C}}
\]

(2)

Where:

- \(C\) = cycle length (seconds),
- \(d'\) = average vehicle delay (seconds/vehicle),
- \(d_i\) = average vehicle delay assuming D/D/1 queue (seconds/vehicle),
- \(g\) = effective green (seconds),
- \(r\) = duration of red interval (seconds),
- \(X\) = degree of saturation = \(\lambda C / \mu g\),
- \(X_i\) = minimum of \((X, 1.0)\),
- \(\lambda\) = average arrival rate (vehicles/second),
- \(\mu\) = average service rate (saturation flow rate) in vehicles/second.
There are three components in Webster’s expression. The first component is the average delay assuming deterministic arrivals and deterministic service rate. The second component accounts for the delay due to the randomness of arrivals and was developed on the basis of steady-state stochastic queuing theory assuming Poisson arrivals and deterministic service. The last component is an empirical correction factor that ranges from 5 to 15% of $d'$. Webster’s original formulation is valid only for $X < 1.0$. To permit application to oversaturated conditions researchers employed the coordinate transformation technique to develop expressions for delay that are applicable even for $X > 1.0$. One such delay expression, incorporated within the HCM, is given by

$$d = d_1(PF) + d_2 + d_3$$  \hspace{1cm} (3)$$

$$PF = \frac{(1-P)f_{PA}}{1 - \left(\frac{g}{C}\right)}$$  \hspace{1cm} (4)$$

$$d_2 = 900T \left[(X-1) + \sqrt{(X-1)^2 + \frac{8klX}{cT}} \right]$$  \hspace{1cm} (2)$$

Where:

$c$ = lane group capacity (veh/h),

$C$ = cycle length (seconds),

$d$ = control delay per vehicle (seconds/veh),

$d_1$ = uniform control delay assuming uniform arrivals (seconds/veh),

$d_3$ = initial queue delay (seconds/veh),

$f_{PA}$ = supplemental adjustment factor for platoon arriving during green,

$g$ = duration of green interval (seconds),

$g/C$ = proportion of green time available,

$k$ = incremental delay based on controller settings,

$l$ = upstream filtering / metering adjustment factor, and

$PF$ = progression adjustment factor

$P$ = proportion of vehicles arriving on green

$T$ = analysis period (hour),

$X$ = lane group $v/c$ ratio or degree of saturation.

The delay expressions in the CCG have a similar basis and are given by
\[ d = k_j d_1 + d_2 \]  

\[ d_2 = 15 t_e \left[ (X - 1) + \sqrt{(X - 1)^2 + \frac{240X}{ct_e}} \right] \]  

Where:

- \( c \) = capacity = \( S(g / C) \),
- \( C \) = cycle length (seconds),
- \( d_1 \) = uniform delay D/D/1 queue, and
- \( d_2 \) = overload delay
- \( g \) = duration of effective green interval (seconds),
- \( k_j \) = progression factor,
- \( S \) = saturation flow rate (vph),
- \( t_e \) = evaluation time in minutes,
- \( X \) = degree of saturation = \( \lambda / C \),

For both the HCM and the CCG expressions, delay is primarily a function of volume and capacity. Volume is typically the hourly flow rate associated with the peak 15-minutes (i.e., volume = peak hour volume/PHF). Capacity is a function of the signal timing and the saturation flow rate.

However, in both methods no consideration is given to the distributions of these inputs and only single point estimates are used.

2.2 Previous Research

Very little research appears to have been conducted specifically investigating day-to-day variability of the inputs to signal delay analysis. One recent relevant study conducted by Sullivan et al., (2006) examined the impact of day to day variations in urban traffic peak hour volumes on intersection service levels. Using weekday data from 22 directional continuous traffic counting stations in the city of Milwaukee, the authors computed the coefficient of variation (COV) of peak hour traffic volume. They found that the COV ranged from 0.048 to 0.155 with a mean of 0.089.

Using this COV in peak hour volumes, they examined the impact on a hypothetical intersection approach controlled by a fixed time signal with a 90 second cycle length and an assumed saturation flow rate of 1,900 vph. The approach delay was estimated using the HCM method for mean peak hour volumes, the 85th percentile volume (i.e. mean plus one standard deviation) and the 97.5 percentile volume (i.e. mean plus two standard deviations). The authors found that the use of average volume to capacity ratio tends to understate level of service at busy intersections and concluded that for intersections operating at LOS D, a 10 % increase in traffic volumes would cause deterioration to LOS E or LOS F, about 15% of the time. The authors also concluded that “it is desirable to base intersection service level computations on several days’ peak hour volumes”. 
However they did not make any recommendations regarding how many days or how this could be computed.

The impact of PHF on estimated intersection performance and the selection of appropriate values of PHF was examined by Tarko et al., (2005) who proposed a prediction model based on time of day, population, rush hour volume and road class.

\[
PHF = 1 - \exp(-2.23 + 0.435AM + 0.209POP - 0.258VOL)
\]  

Where:
- \(PHF\) = peak hour factor,
- \(AM\) = 1; if morning AM; 0 otherwise,
- \(VOL\) = rush hour volume (in thousands/hour),
- \(POP\) = population.

The results obtained using the equation was compared with field results and a standard error of 0.072 was calculated. Tarko’s model can be used by traffic engineers to estimate values of PHF for a given intersection. However, Tarko’s model does not provide any insights to the degree of day to day variation that exists in the PHF at a given intersection.

Kamarajugadda et al., (2003) proposed an analytical method to compute the impact of day-to-day variability of peak hour volume on variance of delay. They used the Taylor series expansion on Equation 2 to approximate delay for under saturated conditions. A distribution was then assumed to estimate the mean delay and confidence interval. The Taylor series expansion was evaluated using Monte Carlo Simulation (MCS) and found to produce similar results, however, the authors note that their derivation is not applicable for degree of saturation (X) greater than 0.7. Unfortunately, it is exactly within this range of operating conditions (e.g. \(X > 0.7\)) that day-to-day variability in peak hour volumes has the largest impact on intersection delay and is of most interest to traffic engineers because for conditions with \(X < 0.7\), delays are typically acceptably low.

3. ESTIMATING PEAK HOUR VOLUME VARIABILITY FROM FIELD DATA

3.1 Data Set

Waterloo and Kitchener are adjacent cities located in south western Ontario, Canada approximately 120 km west of Toronto. The combined population of these two cities is 300,000. The regional government, which is responsible for traffic signal operations within these two cites, operates 16 continuous volume counting loop detector stations located mid-block on major arterial roadways Figure 1.
Vehicle counts are obtained for each lane in both directions and aggregated at 15-minute intervals. Data from these vehicle count stations were obtained for the 2005 calendar year. It is assumed that the volume counts from these stations can be interpreted as the approach volumes at the signalized intersections immediately downstream of the detector stations. This assumption implies that:

1. Any oversaturated conditions that may occur at the downstream signalized intersections do not cause queues to spill over the vehicle count stations for any significant portion of the 15 minute interval.
2. There are no significant mid-block flows (entering or leaving) between the vehicle count station and the downstream signalized intersection.

The individual lane data were aggregated to provide vehicle counts by direction (resulting in 26 directional volume count stations) and were filtered to remove data associated with weekends (i.e. Saturdays and Sunday) and all local and national holidays. This resulted in a maximum of 20,736 fifteen minute volume observations for each volume count station. However, as a result of hardware and communication failures, some stations provided only a portion of these data. Stations with less than 70% data availability (i.e. fewer than 14,515 fifteen minute volume counts) were eliminated from the analysis. The remaining 13 stations exhibited an average annual traffic volume ranging from a low of 8,000 vehicles to 30,000 vehicles per non-holiday week day. For each of the 13 stations, for each day, the traffic count data were examined to determine the PM peak hour volume. As a second quality check, the day-to-day variation in peak hour volume was examined for each of the 13 stations. Three stations exhibited erratic variation indicative of abnormal influences, such as lane closures due to construction during a portion of the year long data collection period. These stations were removed from the data set to avoid biasing the analysis. **Figure 2** illustrates the PM peak hour volume data from one of the three stations which were removed from the analysis.
3.2 Variation in Peak-Hour Volume

Table 1 provides descriptive statistics for the peak hour volumes determined for the remaining 10 volume count stations. The mean peak hour volume varies significantly from one station to the next (i.e. ranging from 594 vph to 1375 vph), however, this variation is attributable to different traffic patterns on different roads, and is not of interest with respect to random day-to-day variations.

Table 1: Peak hour volume descriptive statistics

<table>
<thead>
<tr>
<th>Volume Count Detector Station</th>
<th>62</th>
<th>182-WB</th>
<th>184-WB</th>
<th>184-EB</th>
<th>290-WB</th>
<th>312-NB</th>
<th>313-NB</th>
<th>313-SB</th>
<th>484-NB</th>
<th>484-SB</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>1287</td>
<td>1375</td>
<td>658</td>
<td>594</td>
<td>1282</td>
<td>971</td>
<td>822</td>
<td>855</td>
<td>720</td>
<td>961</td>
<td>952</td>
</tr>
<tr>
<td>Std</td>
<td>69.6</td>
<td>97.6</td>
<td>61.5</td>
<td>54.9</td>
<td>111.5</td>
<td>62.6</td>
<td>52.7</td>
<td>112.3</td>
<td>69.4</td>
<td>106.6</td>
<td>79.9</td>
</tr>
<tr>
<td>COV</td>
<td>0.054</td>
<td>0.071</td>
<td>0.094</td>
<td>0.093</td>
<td>0.087</td>
<td>0.065</td>
<td>0.064</td>
<td>0.131</td>
<td>0.096</td>
<td>0.111</td>
<td>0.087</td>
</tr>
<tr>
<td>Obs.</td>
<td>209</td>
<td>213</td>
<td>213</td>
<td>213</td>
<td>213</td>
<td>214</td>
<td>214</td>
<td>214</td>
<td>204</td>
<td>171</td>
<td>208</td>
</tr>
<tr>
<td>Max</td>
<td>1448</td>
<td>1671</td>
<td>1047</td>
<td>788</td>
<td>1750</td>
<td>1160</td>
<td>987</td>
<td>1033</td>
<td>1134</td>
<td>1193</td>
<td>1750</td>
</tr>
<tr>
<td>Min</td>
<td>1042</td>
<td>811</td>
<td>454</td>
<td>277</td>
<td>996</td>
<td>746</td>
<td>640</td>
<td>564</td>
<td>558</td>
<td>490</td>
<td>277</td>
</tr>
</tbody>
</table>

What is of interest, however, is the day-to-day variation in the peak hour volume that occurs at each site. This variation can be quantified by the coefficient of variation (COV) which is computed as the ratio of the standard deviation over the mean. The COV varies from a minimum of 5.4% to a maximum of 13.1% and on average is equal to 8.7%. These results are very similar to those obtained by Sullivan et al. (2006) using similar data from the City of Milwaukee.

The COV can be used to characterise the variability within the approach volume distribution, however we also are interested to determine the shape of the distribution. This was accomplished by normalizing each peak hour volume observation by dividing it by the mean peak hour volume for that volume count station. Consequently, it was possible to create distributions of normalized peak hour volumes and to compare these distributions for each of the 10 volume count stations (Figure 3).

The Kolmogorov-Smirnov test was used to determine if each distribution could be adequately described by the Normal, Gamma, and Log-Normal distribution at the 99%
level of confidence. It was found that the 10 distributions of day-to-day normalized peak hour volume are best described by the Normal distribution with a mean of 1.0 and a standard deviation of 0.087.
Figure 3: Distributions of normalized peak hour volume
3.3 Correlation of Peak Hour Volumes

In the previous section, it was determined that the day-to-day variation in weekday peak hour volumes can be modelled by a normal distribution with a coefficient of variation of 0.087. However, there remains the question of whether or not the peak hour traffic demands on each intersection approach are statistically correlated. The volume count data represented mid-block flows from various locations throughout Waterloo region. Consequently, it was not possible to directly determine the correlation between traffic volumes on different approaches to the same intersection. Nevertheless, it was possible to test the extent to which peak hour traffic volumes at different mid-block locations are correlated. A high correlation could be interpreted to mean that when peak hour traffic demands are higher than average they tend to be higher than average at all locations including all approaches to an intersection.

The correlation coefficient \( \rho \) was computed between the peak hour volumes for each pair of stations (Table 2). The value of \( \rho \) ranged from 0.003 to 0.55 with an average of 0.3 indicating that in general the level of correlation is relatively weak.

<table>
<thead>
<tr>
<th>Station</th>
<th>62</th>
<th>182-WB</th>
<th>184-WB</th>
<th>184-EB</th>
<th>290-WB</th>
<th>312-NB</th>
<th>313-NB</th>
<th>313-SB</th>
<th>484-NB</th>
<th>484-SB</th>
</tr>
</thead>
<tbody>
<tr>
<td>62</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>182-WB</td>
<td>0.3712</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>184-WB</td>
<td>0.4071</td>
<td>0.0501</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>184-EB</td>
<td>0.3899</td>
<td>0.0892</td>
<td>0.4745</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>290-WB</td>
<td>0.3799</td>
<td>0.2233</td>
<td>0.3058</td>
<td>0.0043</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>312-NB</td>
<td>0.5545</td>
<td>0.3457</td>
<td>0.3624</td>
<td>0.3201</td>
<td>0.4413</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>313-NB</td>
<td>0.4348</td>
<td>0.3018</td>
<td>0.3603</td>
<td>0.5205</td>
<td>0.0199</td>
<td>0.3882</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>313-SB</td>
<td>0.3512</td>
<td>0.5350</td>
<td>0.3574</td>
<td>0.3739</td>
<td>0.3103</td>
<td>0.4513</td>
<td>0.5329</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>484-NB</td>
<td>0.4395</td>
<td>0.1150</td>
<td>0.2560</td>
<td>0.2141</td>
<td>0.3240</td>
<td>0.3232</td>
<td>0.2112</td>
<td>0.1224</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>484-SB</td>
<td>0.3315</td>
<td>0.2362</td>
<td>0.1446</td>
<td>0.2479</td>
<td>0.0032</td>
<td>0.1732</td>
<td>0.3968</td>
<td>0.0670</td>
<td>0.0941</td>
<td>1</td>
</tr>
</tbody>
</table>

This suggests that when peak hour traffic volumes on one approach are much lower (or higher) than average there is not a high likelihood that peak hour volumes on the other approaches are also lower (or higher) than average.

Further work is required to confirm that a similar range of correlation exists between peak hour volumes on different approaches to the same intersection. Nevertheless, the importance of the statistical correlation between approach volumes is demonstrated in section 4 of this paper.

4. VARIABILITY OF DELAY AND LOS

The objective of this section is to explore the impact that the day-to-day variability of peak hour volumes has on the operating characteristics of a typical 4-leg intersection operating under a fixed time traffic signal control strategy. The following section describes the hypothetical intersection. Section 4.2 describes the Monte Carlo simulation used to evaluate the intersection performance. The results of the simulation are presented in section 4.3.
4.1 Hypothetical Intersection

A hypothetical 4-leg intersection was assumed. Each approach consisted of an exclusive left turn lane, an exclusive through lane, and a shared through and right turn lane Figure 4. All lane widths, grade, curb radii, etc. were considered to be ideal with no on-street parking, no transit vehicles, and adequate storage and discharge space.

![Figure 4: Hypothetical intersection for Monte Carlo Simulation](image)

The base (ideal) saturation flow rate was assumed to be 1900 pcppl. The intersection was controlled by a two-phase signal timing plan with a cycle length of 80s; 38s effective green for phase 1; 34s effective green for phase 2; and 4 seconds of intergreen between each phase. Right-turn on red was not permitted.

Six traffic demand scenarios were considered. For each scenario, the turning movement proportions remained constant (1% left turn, 79% through, and 20% right turn) but the total approach demands varied (Table 3).

For each scenario, traffic volumes were selected so that the intersection delay associated with the mean volumes fell within the specified LOS range. For all cases, the traffic stream was assumed to consist of only passenger cars.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Delay (s/veh)</th>
<th>Degree of Saturation (X)</th>
<th>EB</th>
<th>WB</th>
<th>NB</th>
<th>SB</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOS B</td>
<td>15.6</td>
<td>0.601</td>
<td>956</td>
<td>950</td>
<td>852</td>
<td>861</td>
</tr>
<tr>
<td>LOS B/C</td>
<td>24.3</td>
<td>0.877</td>
<td>1390</td>
<td>1381</td>
<td>1239</td>
<td>1252</td>
</tr>
<tr>
<td>LOS C</td>
<td>27.6</td>
<td>0.914</td>
<td>1448</td>
<td>1439</td>
<td>1291</td>
<td>1304</td>
</tr>
<tr>
<td>LOS C/D</td>
<td>40.1</td>
<td>0.985</td>
<td>1564</td>
<td>1554</td>
<td>1394</td>
<td>1408</td>
</tr>
<tr>
<td>LOS D</td>
<td>44.7</td>
<td>1.003</td>
<td>1593</td>
<td>1583</td>
<td>1420</td>
<td>1434</td>
</tr>
<tr>
<td>LOS E</td>
<td>65.4</td>
<td>1.067</td>
<td>1694</td>
<td>1684</td>
<td>1510</td>
<td>1526</td>
</tr>
</tbody>
</table>

1 Delay computing using HCM method and average approach peak hour demands
4.2 Monte Carlo Simulation

The performance of the hypothetical intersection was evaluated using the methodology defined by the HCM. The following parameter values were assumed:

- Evaluation time period = 0.25 hours
- PHF = 0.923
- Area type = 1 (CBD)
- Arrival type = 4

It should be noted that the same procedure could have been followed using the delay estimation methods from the Canadian Capacity Guide or even a software tool such as Synchro. It is anticipated that the results obtained from the use of one of these other delay estimation techniques would not differ significantly from the findings described in this paper.

For each of the 6 demand scenarios, 1000 Monte Carlo trials were evaluated. For each Monte Carlo trial, peak hour approach volumes were generated randomly using a Normal distribution with a COV = 0.087 and the mean peak hour volume from Table 4. This was repeated 4 times, each for a different level of correlation between the approach volumes, namely Uncorrelated ($\rho = 0$); Perfectly Correlated ($\rho = 1.0$); Average Correlation ($\rho = 0.3$); and High Level of Correlation ($\rho = 0.55$). For all simulations, the signal timing plan, saturation flow rate, PHF, and turning proportions and all other inputs except the approach volumes remained unchanged.

4.3 Results

Figure 5 illustrates the cumulative distribution of average intersection vehicle delay estimated by the HCM delay equations for the 6 demand scenarios defined in Table 3 and assuming that approach volumes are not correlated (i.e. $\rho = 0$). Several observations can be made on the basis of the results in Figure 5.

First, as expected, the variation in the intersection performance (i.e. delay) increases dramatically as the degree of saturation increases (i.e. the curves shift to the right). Second, the variation in the intersection delay increases dramatically as the degree of saturation increases. This is reflected by the range of delay values represented by each curve. For example, for $v/c = 0.601$, the cumulative distribution curve is almost a vertical line, implying that almost all 1000 simulation trials resulted in the same intersection delay. Conversely, for $v/c=1.067$, the cumulative distribution curve indicates that some of the trials resulted in delays as low as approximately 35 seconds and some as high as approximately 115 seconds.

The importance of these finding is made clearer by recognising that each Monte Carlo trial can be thought of as a single day. For an intersection operating on average during the peak hour at a $v/c=1.067$, the delay that would estimated using the HCM
methodology could range from LOS C to LOS F depending solely on the day that turning movement counts were obtained.

**Figure 5: Impact of Degree of saturation on Intersection performance ($\rho = 0$)**

![Figure 5: Impact of Degree of saturation on Intersection performance ($\rho = 0$)](image)

The importance of the impact of correlation between peak hour approach volumes on different approaches to the same intersection is illustrated in **Figure 6** which depicts the cumulative distribution of average intersection delay associated with two demand scenarios ($X = 0.914$, and $X = 1.067$). The cumulative delay distributions are shown for the four different levels of correlation.
Figure 6: Distribution of intersection delay for various traffic demands

It can be observed from the figure that the variation in intersection delay increases with increasing correlation of the approach volumes. This impact is increasingly pronounced as the intersection design quality of service decreases (i.e. moves to X = 1.067).

It is also noted from Figure 5 and Figure 6 that the distribution of intersection delay appears to be generally Normally distributed. This was confirmed by the Kolmogorov-
Smirnov which showed that 22 of the 24 cases (i.e. six Degree of Saturation scenarios each at four levels of correlation) could be best described by a Normal distribution. The remaining two cases ($X = 1.067$ with $\rho = 0$ and $X = 1.067$ with $\rho = 0.55$) were best described by the Log-Normal distribution.

**Figure 7** illustrates the impact that the non-linear relationship between volume and delay has on estimating the mean delay. Results show that the difference in delay estimation (Estimation error) as obtained from the mean of the distribution and the delay associated with the mean volumes can be as much as 18%. This implies that ignoring the variability of these volumes, under-estimates the true average intersection delay by as much as 18%.

The preceding discussion has focussed on estimating the distribution of intersection delay given that the true mean peak hour volume or the distribution of peak hour approach volumes is known. Of course in practice, the true mean and the distribution of peak hour volume are rarely known. In current practice, a single observation is obtained and used to determine the signal design. Given the variability in the intersection performance, a single observation is rarely adequate. The obvious question then is how many days of observations are required.

**Figure 8** illustrates the number of days of observations of peak hour volumes that are required to determine the average intersection delay within a given tolerance level (results are provided for $\rho = 0.3$). The data in **Figure 8** were generated using a two stage sampling process where by an initial sample of 3 days of observations of peak hour volume were randomly selected from 1000 days (i.e. the 1000 Monte Carlo simulation trials). For each day’s observation, the intersection delay was computed using the standard HCM method. The mean and sample standard deviation of intersection delay
associated with the three observations was computed. Then the number of observations required to achieve a given level of accuracy in the estimate of the mean delay was computed as

\[ n_2 = \left( \frac{t_{n_2-1, \alpha} \cdot s}{d} \right)^2 \]  

(11)

Where:
- \( n_2 \) = required number of days of observations of peak hour volume
- \( t_{n_2-1, \alpha} \) = student t distribution value for \( n_2 - 1 \) degrees of freedom and a probability of \( \alpha \)
- \( s \) = sample standard deviation of intersection delay computed from the initial sample
- \( d \) = maximum desired error in the estimation of the true mean intersection delay

Values of \( d \) between 2 seconds and 60 seconds were examined. The two stage sampling process was conducted 500 times for each level of \( d \) considered and the average value of \( n_2 \) reported. In Figure 8, \( d \) is represented as a fraction of the true average intersection delay (x-axis).

**Figure 8: Number of days of peak hour volume observations required to achieve specified accuracy of average intersection delay**

The results in Figure 8 can be used to determine the number of days of observations required to achieve a selected maximum estimation error. For example, if a maximum estimation error of 40% of the true mean intersection delay is acceptable, then the number of days of peak hour volumes required is estimated to be 1, 2, 3, 3, 4, and 4, for
the intersection operating at X= 0.601, 0.877, 0.914, 0.985, 1.003 and 1.067 respectively. The associated expected error in the estimated average intersection delay is 5, 10, 12, 19, 21, and 29 seconds respectively. Obviously, if a more accurate estimate of the mean intersection delay is required (e.g. 20% error), then more observations of peak hour volumes must also be made. The choice of the acceptable level of error represents a trade-off between accuracy or reliability of the estimated intersection performance and the cost of acquiring turning movement counts. Current practice typically is to conduct volume counts on a single day, implying reliability of the estimated intersection performance may be very poor. Given that decisions associated with intersection improvements (and possibly developer fees) may be made on these intersection performance estimates, greater reliability of the estimates may be necessary.

4.4 Comparison to Previous Work

The results from the proposed Monte Carlo simulation method were compared with results reported by Kamarajuggada et al. (2003) who used an analytical approach for estimating the variability of intersection delays. In their study, Kamarajuggada evaluated a single lane approach to a signalized intersection with the following characteristics:

- Coefficient of Variation of peak hour volume = 0.25.
- Ideal saturation flow rate = 1800 pcphpl
- g/C ratio = 0.3.
- Mean peak hour volume = 300 pAPH.

We modelled this same situation using Monte Carlo simulation for degrees of saturation ranging from 0.1 to 0.9 in 0.1 increments. For each degree of saturation condition 1000 Monte Carlo simulation trials were conducted.

It is important to note, that the COV=0.25 is much larger than was observed in the empirical traffic count data from Waterloo. It is not clear why Kamarajuggada et al. selected such a large COV. Nevertheless, we used the same value in order to provide a direct comparison.

The results of the comparison are depicted in Figure 9. From these results, it appears that the mean delays from the proposed methodology are consistent with the results obtained from the work of Kamarajuggada. Kamarajuggada et al., did not provide results for X> 0.7 and therefore, no comparison can be made over this range.

It is also interesting to note that the results from Kamarajuggada suggest that the lower bound confidence limit of average delay becomes smaller as X increases (especially for X=0.7). This appears to be counter-intuitive. Conversely, the Monte Carlo simulation results suggest that the upper and lower 95% confidence limits of delay increase as X increases.
Naturally, due to the non-linear relationship between intersection delay and degree of saturation, the upper 95% confidence limit increases at a much greater rate than does the lower 95% confidence limit.

It must also be noted that as these results reflect delay on a single approach, the impact of the correlation of volumes on different approaches is not captured.

**Figure 9: Comparison of proposed methodology with Kamarajugadda work**

The results of this comparison suggest that the proposed Monte Carlo simulation approach and the analytical approach used by Kamarajugadda provide similar estimates for the mean delay for $X \leq 0.7$ and for the 95% confidence limits of delay for $X \leq 0.6$. However, Kamarajugadda suggests that their analytical method is not applicable for $X > 0.7$. Furthermore, their method is not able to consider the correlation that exists between peak hour volumes on different approaches – a factor shown in this study to be important.

5. **CONCLUSIONS AND RECOMMENDATIONS**

The day-to-day variability of peak hour approach volumes are not considered within signal evaluation and design methodologies. Rather, the current practice is to determine intersection performance, in terms of average vehicle delay, on the basis of peak hour volumes observed over a single day.

In this study, we have determined on the basis of empirical data that:
1. The day-to-day variation of weekday peak hour volumes can be represented by a Normal distribution with a coefficient of variation of 0.087. These findings are consistent with the findings of Sullivan et al (2006).

2. The variation of peak hour approach volumes are not statistically independent but appear to exhibit a moderate correlation (mean $\rho = 0.3$).

3. Correlation between the peak hour volumes on each intersection approach impacts the variability of intersection delay. The higher the degree of correlation, the greater the variability in the intersection delays.

4. The estimation of average intersection delay on the basis of average peak hour volumes under-estimated the true delay by as much as 18%. Furthermore, the greatest underestimation error occurs for intersections operating in the range of $X \approx 1$. Depending on the $g/C$ ratio, this can be associated with an intersection LOS D or even C.

5. The number of days of observations of peak hour volumes required to estimate intersection performance was established as a function of the desired level of accuracy.

On the basis of these observations and conclusions, it is recommended that:

1. Additional field data be obtained from another location to confirm the findings of this study.

2. The impact of day-to-day variability of the PHF and turning movement proportions on intersection performance be examined.

3. Criteria be established to incorporate the day-to-day variability of these parameters within existing signalized intersection evaluation and analysis methodologies.

ACKNOWLEDGEMENTS

The authors wish to acknowledge Mark Liddell from the Regional Municipality of Waterloo for providing the system detector data used in this study. The authors also wish to acknowledge the following agencies/organization that provided financial support for this project:

- Transport Canada, Urban Transportation Showcase Program
- Regional Municipality of Waterloo, and
- Natural Science and Engineering Research Council of Canada.

REFERENCES


Highway Capacity Manual (2000) Published by the Transportation Research Board of the National Academies Washington DC.
Kamarajugadda A. and Brian Park (2003), *Stochastic Traffic Signal Timing Optimization*, Final report of ITS Center project: Signal timing algorithm, Center for ITS Implementation Research, U.S. DOT University Transportation Center, Department of Civil Engineering, University of Virginia.


AUTHORS’ INFORMATION

Zeeshan Abdy
PhD Candidate
Department of Civil and Environmental Engineering,
University of Waterloo
Waterloo, Ontario, N2L 3G1, Canada
Tel: (519) 888-4567 ext. 6596
E-mail: zrabdy@uwaterloo.ca

Bruce Hellinga, PhD, PEng
Associate Professor
Department of Civil and Environmental Engineering,
University of Waterloo
Waterloo, Ontario, N2L 3G1, Canada
Tel: (519) 888-4567 ext. 2630
Fax: (519) 888-4349
E-mail: bhellinga@uwaterloo.ca