Reducing Bias in Probe-Based Arterial Link Travel Time Estimates For ITS Applications

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Abstract

The use of probe vehicles to provide estimates of link travel times has been suggested as a means of obtaining travel times within signalized networks for use in advanced traveler information systems (ATIS). Previous research has shown that bias in arrival time distributions of probe vehicles will lead to a systematic bias in the sample estimate of the mean. This paper proposes a methodology for reducing the effect of this bias. The method, based on stratified sampling techniques, requires that vehicle count data be obtained from an in-road loop detector or other traffic surveillance method. The effectiveness of the methodology is illustrated using simulation results for a single intersection approach and for an arterial corridor. The results for the single intersection approach indicate a correlation (R^2) between the biased estimate and the population mean of 0.61, and an improved correlation between the proposed estimation method and the population mean of 0.81. Application of the proposed method to the arterial corridor resulted in a reduction in the mean travel time error of approximately 50%, further indicating that the proposed estimation method provides improved accuracy over the typical method of computing the arithmetic mean of the probe reports.

Introduction

The successful wide scale deployment of Advanced Traveler Information Systems (ATIS) and Advanced Traffic Management Systems (ATMS) depends on the ability to obtain and subsequently disseminate information that accurately reflects network traffic conditions. Many different techniques for assessing traffic conditions have been proposed. However, one method in particular, namely the use of vehicles that are capable of transmitting link travel times to the traffic management center, has received considerable attention. The use of probe vehicles enables a sample of the travel times experienced by all vehicles traversing the link to be obtained. Previous research has examined how accurately probe vehicle travel times (sample) reflect the travel times of all the vehicles (population) that traversed the link (Van Aerde *et al,* 1993; Srinivasan and Jovanis, 1996; Turner and Holder, 1995; Boyce *et al*, 1991a, 1991b). In each case, these researchers assumed that probe reports represent an independent random sample from the traffic stream, and consequently, as the number of probe reports received in a period increases, the sample mean approaches the population mean.

Sen *et al* (1997a, 1997b) examined field data collected from probe vehicles as part of the ADVANCE project. On the basis of a statistical analysis of probe link travel times, they concluded that probe reports are not independent and therefore regardless of the sample size, the sample mean may not approach the population mean.

Recent work by Hellinga and Fu (1999) has demonstrated that the contradictory conclusions reached by Van Aerde *et al*. (1993) and Sen *et al*. (1997a), are indeed both correct but each is appropriate only for specific traffic and sampling conditions. The authors also showed that bias in the probe sample leads to a sample mean that does not asymptotically approach the population mean, regardless of the sample size. As a result of these earlier findings, this paper describes a methodology for reducing the effect that sample bias has on the estimated mean travel time and illustrates the application of this method using simulation data.

The next section describes the development of the method for estimating mean link travel times in which the sample is biased. The effectiveness of this proposed method in reducing bias in the estimated mean travel time is illustrated using simulation data for a single approach to a signalized intersection. Following this, simulation data are used to illustrate the impact of the proposed method for overcoming sample bias on a signalized arterial. Finally conclusions are made regarding the importance of these findings for the design of probe based ATIS and ATMS.

Estimation of Expected Delay

This section proposes an estimator for the expected travel time experienced by a vehicle traversing a signalized arterial link. Previous research (Hellinga an Fu, 1999) has shown that the mean travel time of the probe vehicles (the sub-population from which samples are taken for estimation) is different from the mean travel time of all the vehicles (population) when the probe vehicles represent a biased sample from the population. The bias normally arises when the arrival time distribution of the probes is not consistent with the arrival time distribution of the population. The objective in this section is to develop a methodology for estimating the mean travel times of the population from the probe vehicle travel times in such a way that the impact of any sample bias is removed or at least reduced.

The travel time that a vehicle experiences when traversing a signalized link consists of two components, namely the running time and the delay caused by the signal control. In the following theoretical derivation, we assume that the mean running times of the probe vehicles and the general vehicles are the same and therefore we consider only the difference in mean delay between probes and all vehicles.

Notations

c_y	=	cycle time (seconds)
g	=	effective green interval (seconds)
r	=	effective red interval (seconds)
λ	=	g/c_y
pcu	=	passenger car unit
s	=	saturation flow rate (pcu/second)
c_a	=	capacity (pcu/second), determined by $s\lambda$
q	=	average arrival flow rate during cycle time (pcu/second)
ρ	=	q/s
x	=	degree of saturation during the cycle time, defined as $q/c_a = \rho/\lambda$
P_g	=	proportion of probe vehicles among all vehicles arriving during effective green interval
P_r	=	proportion of probe vehicles among all vehicles arriving during effective red interval
ϕ	=	ratio of the proportion of probe vehicle arrivals during the effective green interval to the proportion of probe vehicle arrivals during the effective red interval, defined as $\phi = P_g/P_r$
δ	=	ratio of estimation error associated with biased approach to proposed approach
i	=	a time interval within the evaluation period
N_i	=	number of general vehicles arriving during interval i
N_T	=	total number of general vehicles arriving during entire evaluation period
n_i	=	number of probe vehicles arriving during interval i
n_p	=	number of probe vehicles arriving during the evaluation period
d_{ji}	=	delay experienced by probe vehicle j in interval i (sec)
d_j	=	delay experienced by the j^{th} probe vehicle during the evaluation period (sec)

D_p = estimate of mean delay of all vehicles computed on the basis of probe information using traditional biased approach (sec)

D'_p = estimate of mean delay of all vehicles computed on the basis of probe information using the proposed approach (sec)

E[D]= expected delay for all vehicles (sec)

E[D_p]= expected delay for probe vehicles (sec)

E[D'_p] = expected delay for all vehicles, determined on the basis of probe vehicle reports (sec)

Traditional Method of Estimating Population Delay

The traditional method of estimating population delay on the basis of probe information is to compute the arithmetic mean probe delay from all probe reports received during the period of interest, and to use this as an estimate of the expected population delay (Equation 1).

$$D_p = \frac{1}{n_p} \sum_{j=1}^{n_p} d_j \qquad (1)$$

While Equation 1 may provide a good estimate (depending on the number of probe reports available) of the expected probe delay, previous research has shown that the expected probe delay does not equal the expected population delay when the arrival time distribution of the probe vehicles is not the same as the arrival time distribution of the population. Thus, while Equation 1 is simple to implement, it will provide biased results under certain conditions, conditions that are likely to occur in practice.

Proposed Method of Estimating Population Delay

We propose the use of stratified sampling for estimating population mean delay on the basis of probe reports. Stratified sampling (e.g. Cochran, 1977) attempts to improve the accuracy of estimates of population characteristics made on the basis of sample statistics, by dividing a heterogeneous population into a number of sub-population, each of which exhibits a greater degree of homogeneity than the population as a whole. Through stratified sampling, small samples from each strata (sub-population) can provide precise estimates of the sub-population characteristics. These individual estimates can be combined to provide precise estimate for the whole population.

In our application, the population consists of all vehicles traversing a link during some time period. The sample is the probe vehicles. We wish to estimate the mean population delay. In applying stratified random sampling, we cannot create strata by delay, since we cannot know the portion of the population that experiences delays associated with a given delay strata. However, delay at a signalized intersection is highly dependent on arrival time, and therefore, we can stratify the population by arrival time to create sub-intervals within each period. The number of vehicles arriving during each sub-interval can be obtained using loop detectors or some other vehicle detection technology. With this knowledge of population arrival time distribution, the estimate of mean population delay is computed as the weighted average of the mean probe delay computed over all sub-intervals. Each probe mean is weighted by the relative frequency of population vehicle arrivals during the interval (stratum weight).

$$D'_p = \sum_{i=1}^{I} \left(\frac{1}{n_p} \sum_{j=1}^{n_p} \left(d_{ji}\right) \cdot \frac{N_i}{N_T} \right) \tag{2}$$

When only a single interval is considered (i.e. I = 1), then Equation 2 reverts to Equation 1 and provides the same bias as does Equation 1. From stratified sampling theory, we know that, if in every stratum (sub-interval) the sample estimate is unbiased, then D'_p is an unbiased estimate of the population delay (D). Unfortunately, in practice we cannot be certain that the probe mean in each sub-interval is an unbiased estimate. Never-the-less, we contend that if I is greater than 1, then on average, Equation 2 will provide a more accurate (less biased) estimate of the population mean delay than will Equation 1. This is proved analytically in the following section for a specific case.

Analytical Comparison of Biased and Proposed Estimation Methods

Consider an idealized single-lane approach that is controlled by a traffic signal operating with a fixed time signal plan. All vehicles are passenger cars and only through movements are permitted at the signal. Vehicle arrivals and departures are deterministic. The population arrival rate is constant and equal to q. Probe arrival rate during the red interval is equal to $P_r q$ and during the green interval is equal to $P_g q$.

For a randomly selected vehicle, its expected delay can be determined from Equation 3, the well known expression for uniform delay (see for example, Hurdle, 1984).

$$E[D] = \frac{r^2}{2c_y}\left(\frac{1}{1-\rho}\right) \tag{3}$$

As shown in Hellinga and Fu (1999), the expected delay for probe vehicles, E[D_p], is not equal to the expected delay of the population (E[D]) as shown in Equation 4.

$$E[D_p] = \frac{r^2}{2c_y}\left(\frac{1}{1-\rho}\right)\left[\frac{1+x^2\lambda^2(\phi-1)}{1+\lambda(\phi-1)}\right] = E[D]\cdot\left[\frac{1+x^2\lambda^2(\phi-1)}{1+\lambda(\phi-1)}\right] \tag{4}$$

As illustrated in Equation 4, the extent to which E[D_p] differs from E[D] depends on, among other factors, the extent to which the probe arrival distribution differs from the arrival distribution of the population. When the arrival distributions are the same (i.e. ϕ = 1.0), then no bias exists and E[D_p] = E[D].

Now if we divide the cycle length into three intervals (strata) as illustrated in Figure 1 (i = 1: $0 < t \le r/2$; i = 2: $r/2 < t \le t_c$; i = 3: $t_c < t \le c_y$), then following the derivation provided in Hellinga and Fu (1999), the expected delay of a vehicle within each interval can be obtained on the basis of a deterministic queuing model (Equation 5).

$$\begin{aligned} &E[D_1] = \frac{(3+\rho)r}{4} && 0 < t < r/2 \\ &E[D_2] = \frac{(3\rho+1)(1-\rho)+4\rho^2\phi}{4(1-\rho+2\rho\phi)} && r/2 < t < t_c \\ &E[D_3] = 0 && t_c < t < c_y \end{aligned} \tag{5}$$

The expected delay for the population based on probe information and the population arrival distribution for each interval, can then be determined by substituting Equation 5 into Equation 2 to produce Equation 6.

$$\begin{aligned} E[D_p'] &= \frac{E[D_1]\times N_1 + E[D_2]\times N_2 + E[D_3]\times N_3}{N_T} \\ &= \frac{E[D_1]\times\left(\frac{1}{2}qr\right) + E[D_2]\times\left(qt_c - \frac{1}{2}qr\right) + E[D_3]\times\left(qc_y - qt_c\right)}{qc_y} \\ &= E[D]\cdot\frac{2+(3\phi-1)\rho+(\phi-1)\rho}{2(1-\rho+2\phi\rho)} \end{aligned} \tag{6}$$

We now show that the estimate based on three intervals (Equation 6) is always better than the estimate based on a single interval (Equation 4). To do so we define the estimate error ratio of these two methods as follows.

$$\delta = \left|\frac{E[D_p]-E[D]}{E[D_p']-E[D]}\right| = \left|\frac{(\rho^2-\lambda)(1-\rho+2\phi\rho)}{\rho(\rho^2-1)(1-\lambda+\phi\lambda)}\right| \tag{7}$$

Based on the conditions $0 \le \rho \le 1.0$, $0 \le \lambda \le 1.0$, $x=\rho/\lambda \le 1.0$ and $\rho \le x$, the error ratio can be reduced to

$$\delta = 2\left|\frac{(\rho^2-\lambda)(1-\rho+2\phi\rho)}{\rho(\rho^2-1)(1-\lambda+\phi\lambda)}\right| = 2\left|\frac{(\lambda-\rho^2)(1-\rho+2\phi\rho)}{\rho(1-\rho^2)(1-\lambda+\phi\lambda)}\right| = 2\left|\frac{(1-x\rho)(1-\rho+2\phi\rho)}{x(1-\rho^2)(1-\lambda+\phi\lambda)}\right| \tag{8}$$

If we substitute x = 1 into the numerator, then we can rewrite the equality of Equation 8 as

$$\delta \geq 2\left|\frac{(1-1\cdot\rho)(1-\rho+2\phi\rho)}{x(1-\rho)(1+\rho)(1-\lambda+\phi\lambda)}\right| = 2\left|\frac{(1-\rho+2\phi\rho)}{x(1+\rho)(1-\lambda+\phi\lambda)}\right| \tag{9}$$

Equation 9 can be further simplified by substituting ρ = 1 into the first term of the denominator.

$$\delta \geq 2\left|\frac{(1-\rho+2\phi\rho)}{x(1+1)(1-\lambda+\phi\lambda)}\right| = \left|\frac{(1-\rho+2\phi\rho)}{x(1-\lambda+\phi\lambda)}\right| = \left|\frac{(1-\rho+\phi\rho)}{(x-\rho+\phi\rho)} + \frac{\phi\rho}{(x-\rho+\phi\rho)}\right| \tag{10}$$

The first term of Equation 10 can be shown to be always greater than or equal to 1 as $x \leq 1$. In the second term, $x \geq \rho$, and therefore $0 \leq \frac{\phi\rho}{(x-\rho+\phi\rho)} \leq 1.0$. Therefore, Equation 10 can be rewritten as

$$\delta \geq |1+0| = 1 \tag{11}$$

Thus, we have shown that for this idealized deterministic situation, regardless of the degree of bias, Equation 2 provides a smaller bias (though not necessarily a bias of zero) in the estimation of the expected population mean delay than does Equation 1. It must be noted that in practice, Equation 2 can only be used to estimate the population mean delay if at least one probe report (d_{ji}) is available for each interval *i*.

It must also be noted that in this derivation, probes represent a random unbiased sample only in intervals 1 and 3. In interval 2, probes represent a biased sample as the arrival distribution of probes is different from the arrival distribution of the population. Therefore, in this application, the use of stratified sampling is not guaranteed to provide an unbiased estimate.

The analytical method described above could be applied for some general number of intervals (I > 1), and for non-uniform arrivals, however, the algebraic expressions become too cumbersome to be of much value. Therefore, the next section uses simulation to quantify the potential improvement in estimation accuracy of applying Equation 2 instead of Equation 1 to a signalized intersection approach.

Application to a Single Intersection Approach

The previous section has demonstrated that when probes represent a biased sample of the population of vehicles (i.e. probe arrival time distribution is not equal to the arrival time distribution of all vehicles), then a reduction in estimation bias can be obtained by considering the arrival time distribution of all vehicles. In the previous section it was shown that for a specific case (i.e. deterministic uniform arrivals), the use of Equation 2 will always provide less (or equal) bias in the estimate of the population delay. However, in this previous application, sampling error was not considered, and it was assumed that vehicle arrivals were deterministic. In this section, we apply Equation 2 to a single intersection approach having random arrivals and attempt to estimate mean population delay over a 5-minute period.

A discrete cycle-by-cycle simulation model was developed to produce data on which to test the relative accuracy of the traditional (Equation 1) and proposed (Equation 2) delay estimation expressions.

The simulation model explicitly models the delay that a vehicle experiences when traversing a two-lane signalised intersection approach. The approach is used exclusively for through traffic and controlled by a pre-timed traffic signal. The vehicle arrivals are randomly distributed with the vehicle headway following a shifted negative exponential distribution with a minimum headway equal to 0.5 seconds. All vehicles are passenger car units.

The vehicle discharge pattern during the green interval depends on the queue status at the approach. If there is no queue present when a vehicle arrives, then the vehicle can be discharged immediately without any delay. Otherwise, the vehicle must wait until the queued vehicles ahead of it are discharged. The saturation flow rate is assumed to be constant at 1800 pcu/h for each lane, which corresponds to a discharge headway of one second for the link. A cycle length of 100 seconds was used.

For each execution of the simulation model, the delay experienced by all vehicles arriving during a 5-minute period was recorded. The start of the 5-minute recording period was randomly selected between time zero and the cycle length, such that the start of the recording period did not always correspond to the start of a cycle. The average delay of all vehicles arriving during the 5-minute period provided the mean population delay (E[D]). The cycle length was divided into only two intervals, the first corresponding to the effective green period (i = 1) and the second to the effective red period (i = 2). The mean of all probe vehicles arriving during the green interval (d_1) provided an estimate of the population mean for this interval (E[D_1]), and d_2 provided an estimate of E[D_2]. The number of general vehicles arriving during each period (N_1 and N_2) was also recorded. Using these recorded values, Equations 1 and 2 were used to estimate the population mean delay.

A total of 7500 five-minute periods were simulated representing 375 combinations of signal and probe parameter combinations. The set of green interval duration to cycle length ratios consisted of three values (g/c_y = {0.3, 0.5, 0.7}), and five degrees of saturation were considered (x = {0.5, 0.6, 0.7, 0.8, 0.9}). Five values were examined for the proportion of probe vehicles within the green and red intervals (P_g = {0.025, 0.05, 0.075, 0.1, 0.125}; P_r = {0.025, 0.05, 0.075, 0.1, 0.125}). For each combination of signal control parameters, the model was executed 20 times in order to capture the stochastic variation.

The results presented in Figures 2 and 3 indicate that even when only very few intervals are selected (i.e. 2), on average the proposed estimation method (Equation 2) provides a better estimate of the mean population delay (R^2 = 0.807) than does Equation 1 (R^2 = 0.607).

Figure 4 illustrates the distribution of delay estimation error resulting from the use of the traditional (Equation 1) and proposed (Equation 2) estimation methods. Error is determined as the difference between the estimate and the true population delay as a proportion of the population delay. The error distribution further indicates the superiority of the proposed estimate over the biased estimate. The errors associated with the proposed method are more closely centered about zero, they exhibit a smaller variance, and the distribution is not skewed.

Statistical testing was conducted to determine if the mean biased (Equation 1) and proposed (Equation 2) estimation errors were significantly different from zero. For each test, the null hypothesis was that the mean estimation error was equal to zero. The results of these tests, which are provided in Table 1, indicate that the null hypothesis is accepted for the proposed estimation method, but cannot be accepted for the biased estimation method. This implies that although the proposed estimation method (Equation 2) results in delay estimates that are subject to sampling errors, the errors are not systematically biased and the mean of the error is not statistically different from zero. The same cannot be said of the biased estimation method (Equation 1) as the results indicate a systematic bias in the estimates.

While the previous results have indicated that the proposed estimation method (Equation 2) provides better delay estimates on average than does the biased method (Equation 1), it is also of value to examine the frequency distribution of cases in which the error associated with the proposed method was smaller than the error associated with the biased method. Figure 5 illustrates the cumulative frequency distribution of the absolute difference between the estimation error of the proposed method and the estimation error of the biased method, as a fraction of the population delay. Thus, positive values of error correspond to cases in which proposed method resulted in a larger absolute estimation error than did the biased method. Conversely, negative error values correspond to cases when the absolute estimation error was smaller for the proposed method than the biased method. Figure 5 indicates that on average the absolute estimation error associated with the proposed method, measured as a function of the population delay, was 16% smaller than the error associated with the biased method. For over 70% of the cases examined, the proposed estimation method provided a smaller absolute estimation error than the biased method. Furthermore, in over

35% of the cases examined, the improvement in the accuracy of the delay estimate provided by the proposed method, was in excess of 20% of the population delay. Thus, it would appear that on the basis of the cases examined, the proposed estimation method provides benefits over the biased method in terms of the accuracy of the estimate of the population delay.

The next section demonstrates the impact of using the proposed method to estimate mean link travel times for an arterial corridor.

Application to an Arterial Corridor

To illustrate the potential improvement in link travel time estimates that may be obtaining by using the proposed estimation method (Equation 2) instead of the biased estimation method (Equation 1), both methods were applied to a simple linear arterial corridor. This application differs from the single intersection approach application described in the previous section in that the arrival distribution of probe vehicles is not explicitly specified, rather the proportion of probes travelling between each origin-destination pair is defined. The arrival distribution is then dependent on the proportion of probes on each O-D path and the turning movements these probes need to make to access the link being examined.

Network Description

The probe data and population data were obtained by modeling the network using the INTEGRATION traffic simulation model (Van Aerde et al, 1996). The network, illustrated in Figure 6, consists of a single arterial roadway that is intersected by two cross streets. Each intersection is controlled by a three-phase fixed-time signal having a cycle length of 120 seconds. Right-turns-on-red are permitted. The phasing scheme, green interval duration and the offset are presented in Figure 6.

The network is modeled for 2.5 hours with time varying demands. Vehicles are generated at all origin zones with negative exponentially distributed headways. The O-D traffic demands between each of the 6 zones and the temporal variation are provided in Table 2.

The time to traverse each link segment, the unique vehicle ID number, the time when the vehicle departed the link (i.e. time of probe report), and the vehicle's origin and destination were recorded for each vehicle. This log represented the travel times experienced by the entire vehicle population.

As demonstrated earlier in this paper, bias in probe estimates result when the arrival distribution and consequently the delay, of the probe vehicles is not representative of the arrival distribution of the population of vehicles. To demonstrate the potential advantages of using the proposed method (Equation 2), we consider a scenario in which the level of market penetration (LMP) of probe vehicles traversing segment 3 is biased with respect to arrival times. The LMP of vehicles from origin 1 that enter segment 3 (i.e. vehicles that make a through movement at intersection *A*) is chosen to be 5%. The LMP for vehicles from origin 6 (i.e. those making a left-turn at intersection *A*) and from origin 2 (i.e. those making a right-turn at intersection *A*) entering segment 3 is chosen to be 25%. Since segment travel time is also a function of the turning movement used to exit the segment, only those vehicles traversing segment 3 that make a through movement at intersection *B* are considered within this example.

Travel time estimates are made for 5-minute periods (30 periods in the 2.5 hour simulation). To account for the randomness in the selection of vehicles as probes, the estimation process was repeated 5 times, each time with a new random sample of probe vehicles. Thus, a total of 150 (5 repetitions $\times$ 30 periods) estimates of the average vehicle travel time were made.

Each 5-minute period was divided into a number of intervals, *I*, for application of Equation 2. Intervals were selected such that exactly 1 probe vehicle appeared within each interval. Time boundaries between intervals were determined as the midpoint between successive probe arrivals. Figure 7 illustrates recorded vehicle travel times as a function of vehicle time of arrival on segment 3 for vehicles associated with period 7. As illustrated in the figure, 7 probe vehicles have exited the segment during the 7th time period and therefore, 7 intervals are used for calculating the travel time estimate using Equation 2. The interval boundaries are illustrated in Figure 7.

Figure 7 illustrates some vehicles with negative arrival times, implying that while the vehicles departed the segment during the seventh 5-minute period, they actually entered segment 3 prior to the beginning of this period.

Table 3 illustrates the application of Equations 1 and 2 for the data illustrated in Figure 7 to estimate the population mean travel times using the biased and proposed method. As indicated in Table 3, each interval contains a single probe vehicle report. When estimating the travel time using the proposed method (Column *A*), each probe travel time is weighted by the fraction of the number of general vehicles detected to enter the segment during the interval (N_i) divided by the total number of vehicles detected to enter the segment during all of the intervals (N_T). Thus, a probe travel time associated with an interval during which a large portion of the general population vehicles arrived (e.g. interval 1), has a much higher weighting than does a probe travel time associated with an interval during which a small proportion of the vehicles arrived (e.g. interval 3). Conversely, the biased estimate (column *B*) applies a constant weighting of $1/n_p$ regardless of the population arrival time distribution.

Aggregate Estimation Results

Table 4 provides the aggregate results for the population, proposed, and biased estimates of mean segment travel time for all 150 estimation periods. An average 5-minute mean segment travel time of 58.4 seconds is obtained using the biased estimation method, representing an average error of (58.4 - 51.0) 7.4 seconds. Conversely, when the proposed method is used, the average error is only 3.7 seconds, representing a 50% reduction in estimation error ((7.4 - 3.7)/7.4). These results support the conclusion made earlier that the proposed method provides a more accurate estimate of mean segment travel times than does a simple arithmetic mean of the probe reports.

On the basis of sampling theory, it is expected that as the number of probe reports during a 5-minute period increases, the accuracy of the estimated mean travel time will also increase. However, it has also been shown earlier in this paper that if a bias in sampling exists, then the biased method of estimating mean link travel times (Equation 1) does not converge to the population mean travel time, even when the proportion of probes is very high. This is illustrated in Figure 8, which depicts the mean estimation error, measured as the absolute difference between the estimated mean and the population mean, divided by the population mean, as a function of the number of probe reports received during the 5-minute period. Results do not consider the 2 periods in which fewer than 2 probe reports were received. Although substantial variation exists in the data, a general trend of decreasing error with increasing number of probes is evident. An exponential regression model was fit to the data from the proposed and biased methods. The regression models indicate that for the network, traffic, and sampling conditions examined, when very few probe reports are received (e.g. less than 5) during a 5-minute period, the errors associated with the biased and proposed methods are approximately equal and are greater than 15% of the true population mean travel time. However, as the number of probe reports increase, the error associated with the proposed estimation method reduces at a faster rate than does the error associated with the biased estimation method.

The importance of these findings is not in the absolute magnitude of the errors (or the coefficients of the regression equations), but rather that the results further support the conclusions that the proposed method of estimating mean link travel times provides an improvement over the biased method that is most often used, in which the arithmetic mean of the probe reports is calculated.

Sensitivity to Measurement Error

The previous analysis has assumed that no measurement error exists in the probe travel time reports. It is expected that if significant measurement error exists, then the use of only a single probe report as an estimate of the population mean for an interval will result in substantial sampling error. The robustness of the proposed estimation technique with respect to measurement error was tested by adding a random error term to each probe report used in the previous analysis. The error term was normally distributed with a mean of zero and a known standard deviation, σ. Equations 1 and 2 were applied to these data as before. Eight levels of error were considered such that the coefficient of variation (standard deviation/mean) was approximately = (0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.4, and 0.6). All other aspects of the data and the application of the estimation methods remained the same as previously. Figure 9 illustrates the average relative estimation error as a function of the coefficient of variation (COV) of the measurement error of the probe travel times. As expected, the proposed method provides a smaller average estimation error when the COV of probe measurement error is equal to zero. However, Figure 9 also indicates that the error associated with the proposed method remains smaller than the error associated with the biased method until the coefficient of variation of measurement error approaches 0.37. The average probe travel time is approximately 57 seconds. Thus, if travel times are normally distributed, then the standard deviation of probe travel times, resulting from measurement error, would be equal to 21 seconds. It would seem highly unlikely that a probe based data collection system, implemented in the field, would provide such a high level of measurement error. It would be far more reasonable to assume that the system measurement errors would be associated with a COV in the range of 0.1. Regardless, the results in Figure 9 indicate that the proposed estimation method remains superior to the biased method (Equation 1) over a wide range of measurement error, up to and exceeding the range of error likely to be encountered within field applications.

Conclusions and Recommendations

The successful wide-scale deployment of ATIS and ATMS requires the capability to obtain accurate estimates of travel times over freeway and arterial roadway segments. The use of probe vehicles provides an opportunity to obtain individual vehicle travel times, however, these probes represent only a sample of all vehicles traversing the roadway segment.

Arterial roadway segments are generally controlled by traffic signals at intersections. The variability in travel time that vehicles experience on an arterial roadway segment is largely determined by the amount of delay experienced at the downstream signal. Delay at signals is largely a function of the time of arrival with respect to the signal cycle. Thus, if the probe vehicles represent a biased sample with respect to their arrival time distribution, then even when data are available from many probes, the mean probe travel time will not tend to the population travel time.

This paper has described another method of estimating population mean travel times even when bias exists in the arrival time distribution. This method used the arrival time distribution of the all vehicles (obtained from loop detectors or some other traffic surveillance method) to weight each probe travel time report. On the basis of the simulation data, this method was shown to be more accurate than the biased method. While the proposed method does not remove all error associated with sampling bias, it is likely to be easy to implement in field conditions, and represents an improvement over biased methods that are typically used. Furthermore, the method has been shown to be superior to the traditional biased methods for the measurement error ranges that could be expected within field implemented systems.

It is recommended that analytical expressions be developed that approximate the error associated with the proposed and biased estimation methods as a function of signal timings, link characteristics, traffic demands, estimation period duration, LMP and LMP bias. These expressions could then be used to quantify the error associated with specific link estimates, and also be used to estimate the number of probe reports required to achieve a desired level of accuracy in travel time estimates.

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Table 1: Testing significance of means of estimated delay

	Biased Method (Equation 1)	Proposed Method (Equation 2)
Null Hypothesis (H_0)	Mean Error = 0	Mean Error = 0
Num. Observations [a]	5916	5916
Mean Error [b]	0.0761	-0.0018
Standard Deviation (sec)	0.4700	0.2467
$Z_{calculated}$	12.45	-0.5612
$Z_{critical\ (95\%\ Confidence\ Limit)}$	1.96	1.96
Outcome	Cannot Accept H_0	Accept H_0

[a] A total of 1584 periods did not have at least 1 probe report during both the red and green interval, and therefore, estimates using the proposed method could not be made.
[b] Error = (Estimate - Population)/Population

Table 2: O-D traffic demands for test network

Base OD Demand (vph)							Temporal Variation		
Origin Zone	Destination Zone						Period	Time (min.)	Proportion of Base Demand
	1	2	3	4	5	6			
1		300	162	808	81	100	1	0 - 30	0.8
2	30		31	154	15	600	2	30 - 60	1.2
3	19	3		100	200	3	3	60 -90	1.6
4	920	120	150		200	160	4	90- 120	1.5
5	192	25	500	75		33	5	120 - 150	1.3
6	250	400	4	19	2				

Table 3: Sample calculation of mean travel time using biased and proposed estimation methods

Interval	Probe Travel Time (Sec)	Number of Vehicles	Proposed A	Biased B
1	40.2	23	13.4	5.7
2	80.4	4	4.7	11.5
3	77.3	3	2.2	11.0
4	75.8	6	5.5	10.8
5	47.8	13	10.4	6.8
6	37.9	10	5.5	5.4
7	77.1	10	11.2	11.0
Total		69	53.7	62.4

[A] Probe Travel Time $\times N_i/N_T$
[B] Probe Travel Time $\times$ 1/7

Table 4: Aggregate results for the estimation of mean 5-minute segment travel times for the arterial corridor application

	Population	Proposed Method	Biased Method
Mean (sec)	51.0	54.7	58.4
Standard Deviation (sec)	5.33	9.01	9.44
Maximum (sec)	63.80	89.23	86.69
Minimum (sec)	43.13	38.53	38.30
Number of 5-minute periods	150	150	150

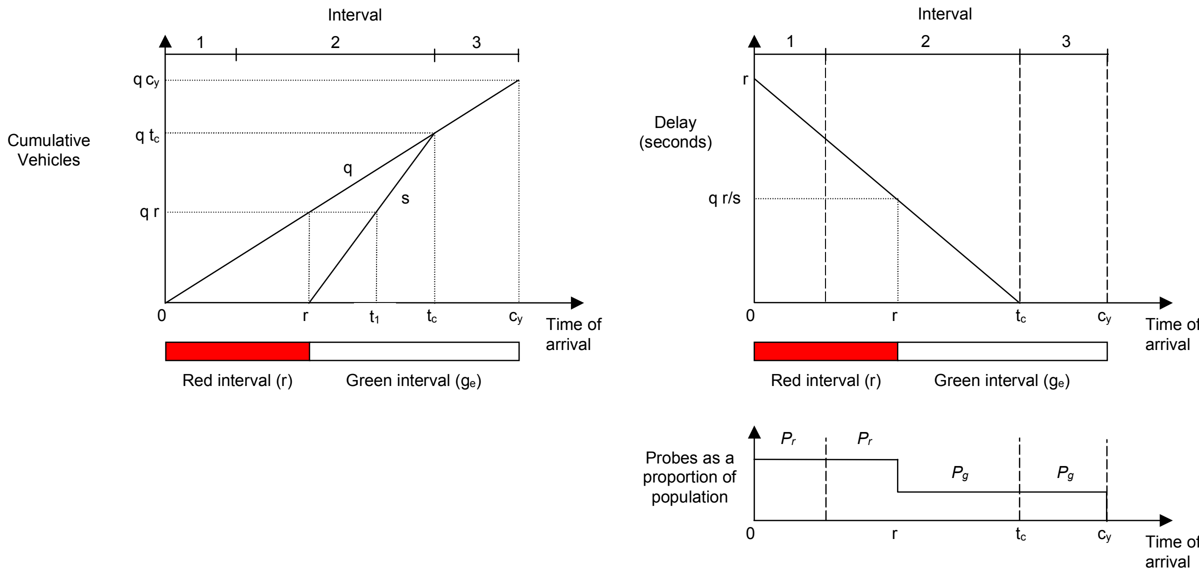

Figure 1: Deterministic queuing diagram

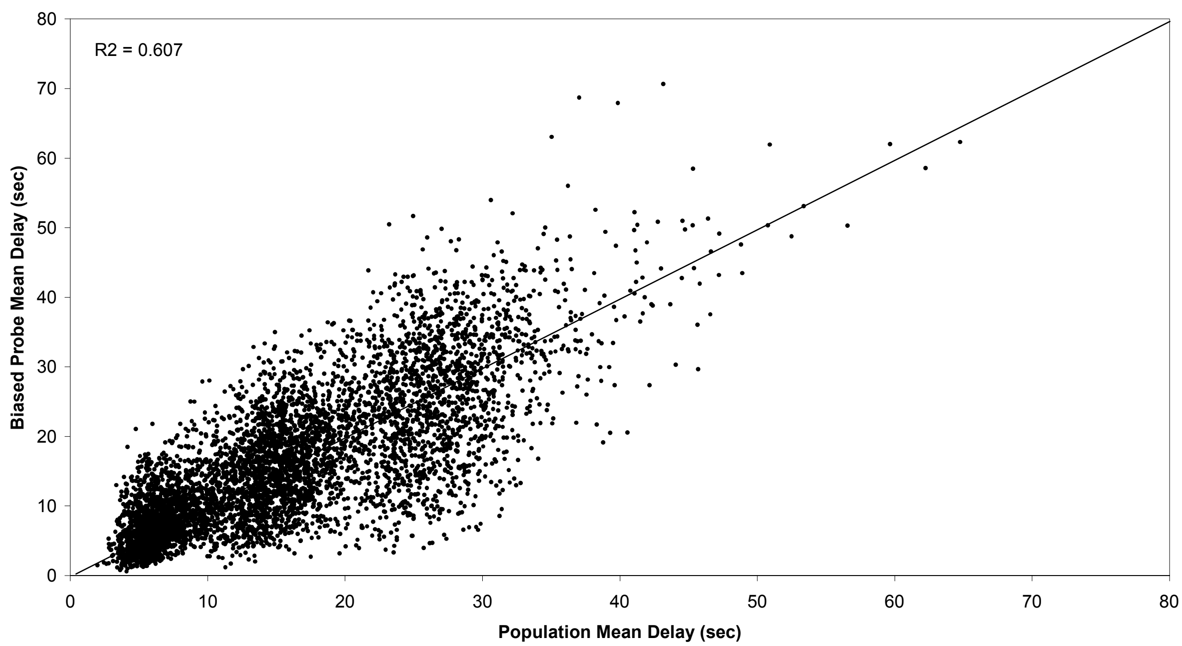

Figure 2: Correlation of biased delay estimate (Equation 1) with population mean

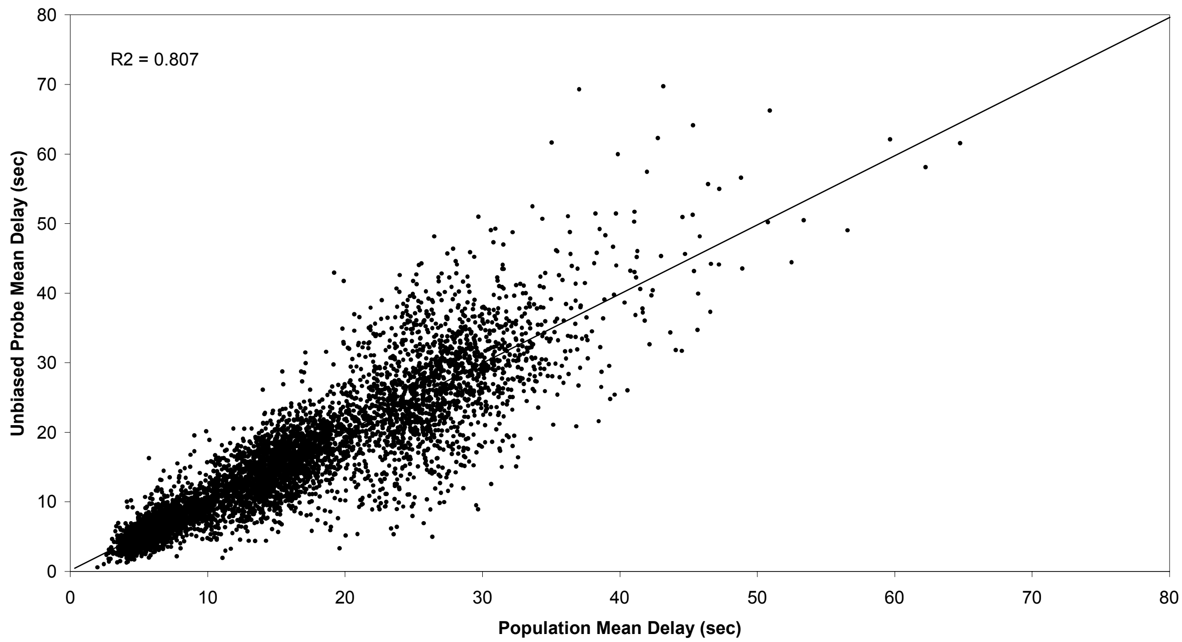

Figure 3: Correlation of proposed delay estimate (Equation 2) with population mean

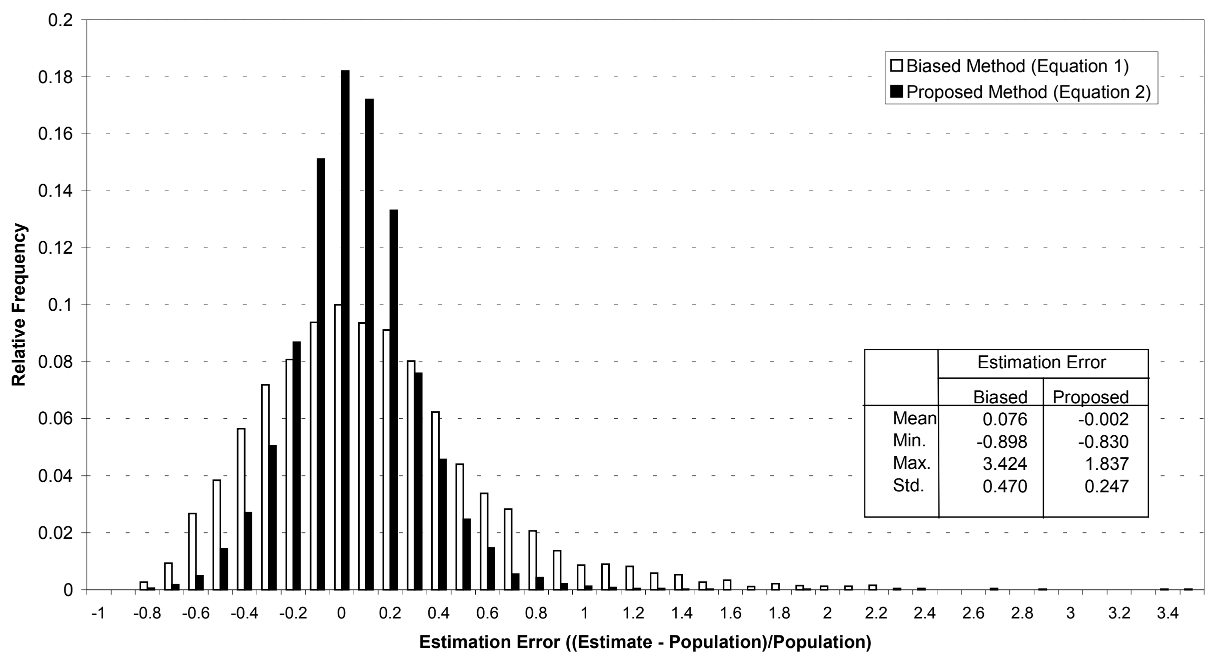

Figure 4: Distribution of delay estimation errors

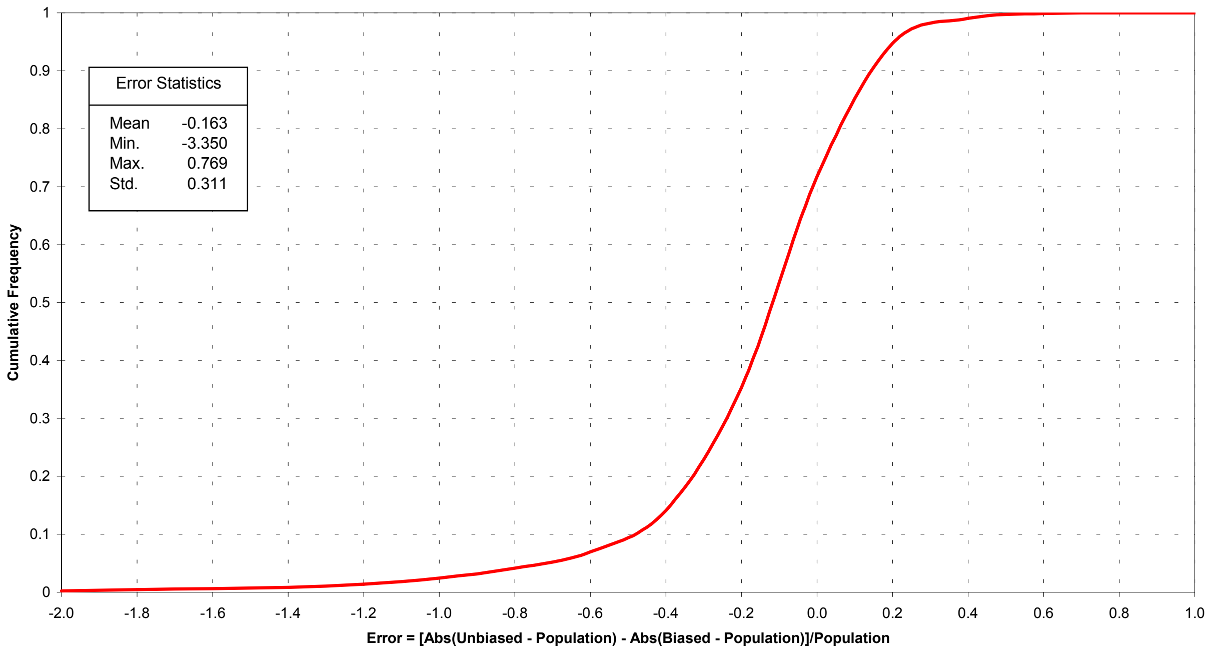

Figure 5: Distribution of estimation errors as a proportion of the population delay

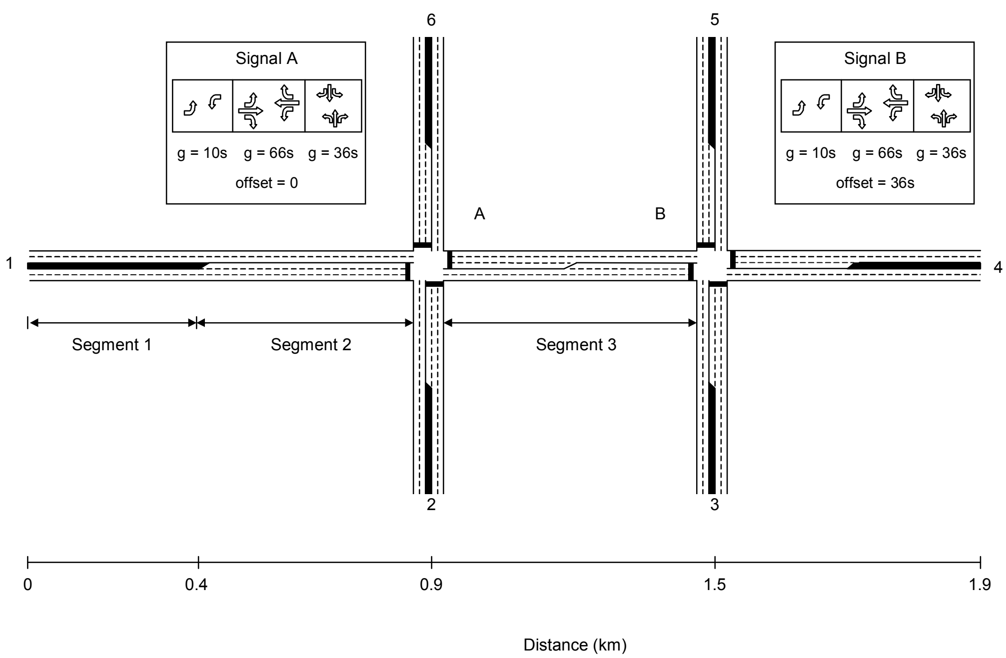

Figure 6: Arterial test network configuration

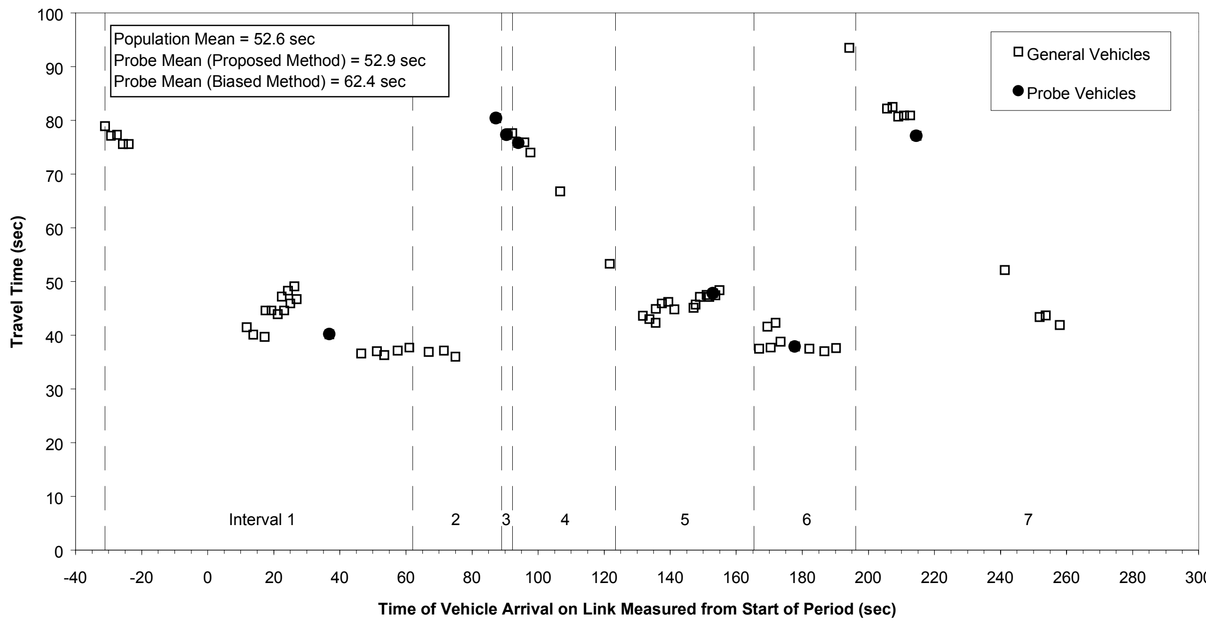

Figure 7: Sample vehicle travel time data as a function of arrival time on link

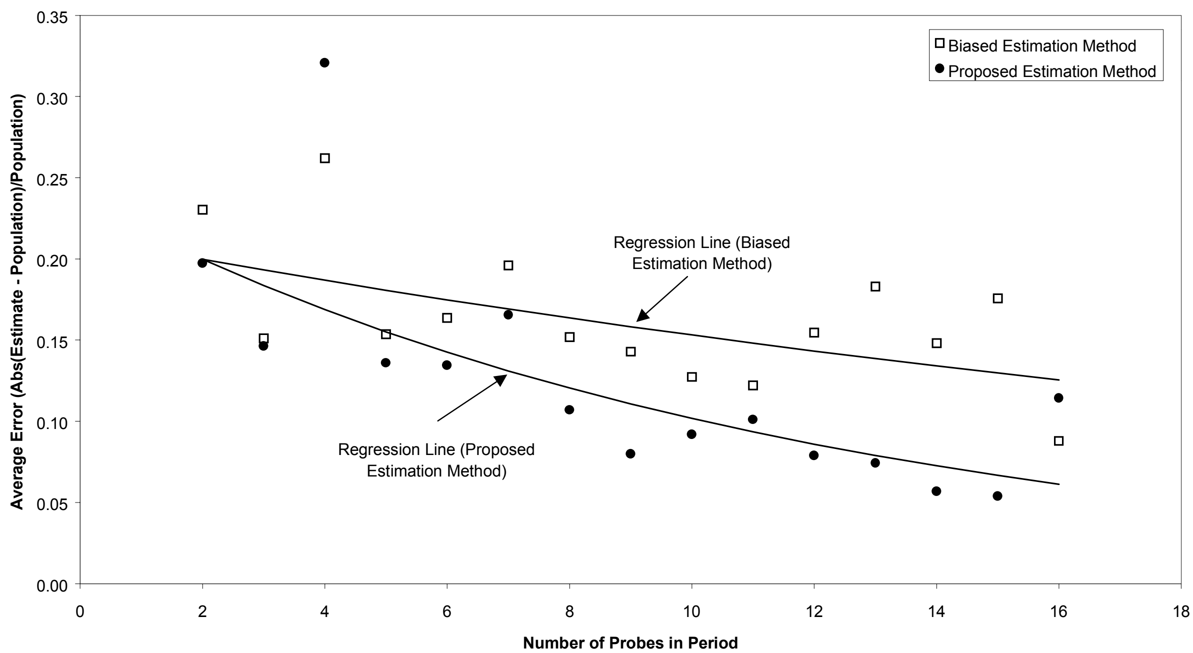

Figure 8: Mean estimation error as a function of number of probe vehicle reports

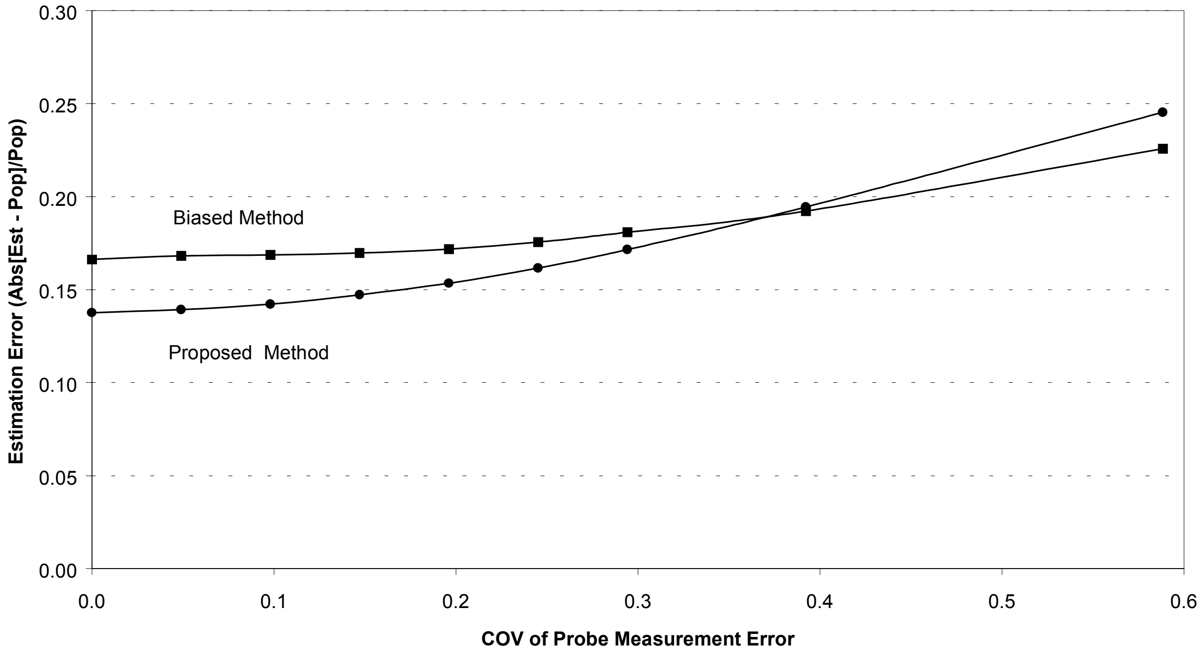

Figure 9: Estimation error as a function of probe measurement error