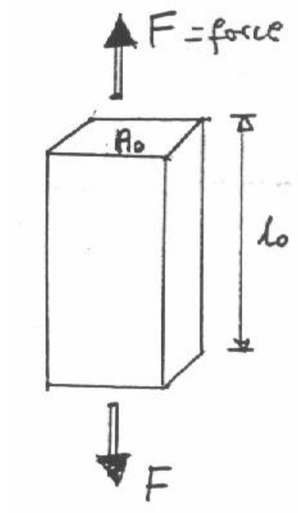
Stress – Strain Relationships



Axial **Stress**:

Axial stress is the force per unit area.

Nominal stress $\sigma = \dfrac{F}{A_o}$ (Engineering Stress)

Where A_o = original cross-sectional area

Unit – Mpa = Megapascals = 10^6 N/m^2

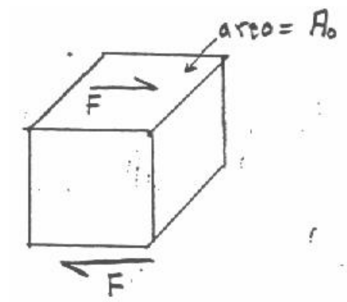
Axial **Strain**:

Engineering strain is defined as the change in length per unit of original length.

Eng. Strain $\varepsilon = \dfrac{l_i - l_o}{l_o}$

Where l_o = original length, l_i = instantaneous length under the load

Unit – None

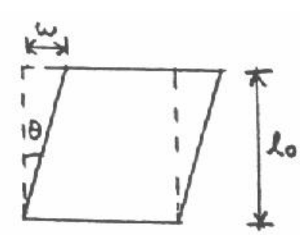


Shear Stress

Shear Stress $\tau = \dfrac{F}{A_o}$

Where F=shear force, A_o = original cross-sectional area

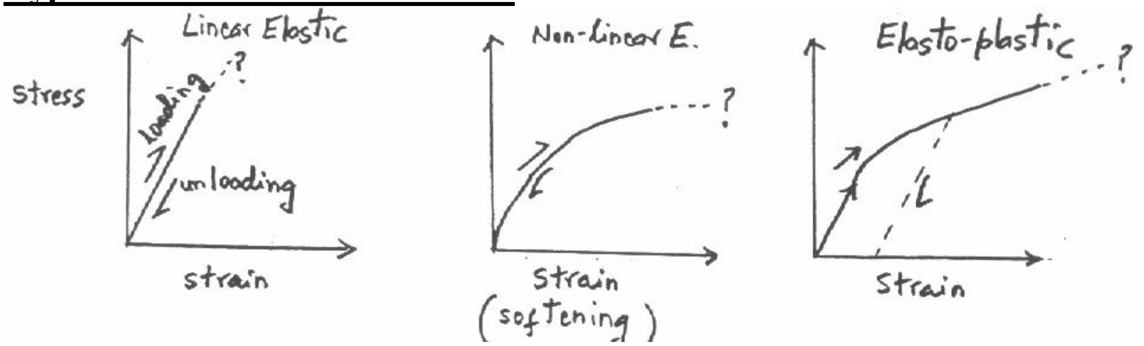
Shear Strain



Shear Strain $\gamma = \tan\theta = \dfrac{w}{l_o}$

Where w and l_o are shown on the diagram

Types of Stress-Strain Relations

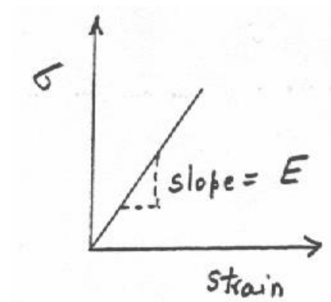


Elastic Material Behaviour

- Strain depends on the magnitude of applied stress and the internal structure of the material
- **Hooke's Law (1678)**
  "as the extension, so the force"
  stress (σ) α strain (ε)

  $$\sigma = E\varepsilon$$

  Where E =Modulus of Elasticity or Young's Modulus (slope of the elastic region of the σ-ε curve)
  E depends on the type of material



- Deformation in which stress is proportional to strain is referred to as Elastic Behaviour of the material
- The greater the value of E, the stiffer the material, or the smaller the elastic strain under an applied stress.
- Elastic Deformation is not permanent. When the applied load is released, the material returns to its original shape.

Shear Stress-Strain

- For elastic material, it is similar to Hooke's Law

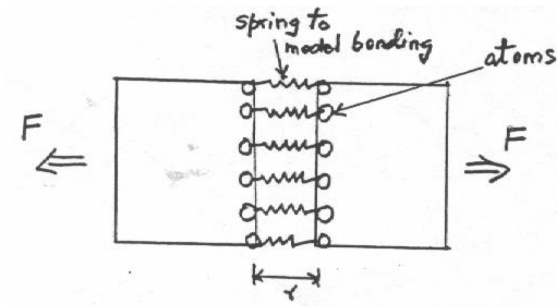
  $$\tau = G\gamma$$

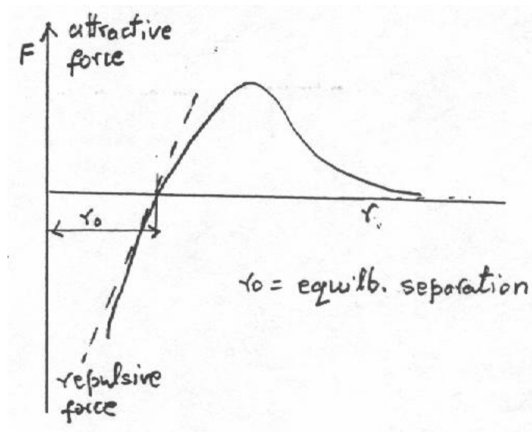
  Where τ = shear stress, γ = shear strain, G = Shear Modulus

Metal/Alloy	E (GPa)	G (GPa)	Poisson's Ratio (υ)
Aluminum	69	25	0.33
Copper	110	46	0.34
Steel	207	83	0.30
Tungsten	407	160	0.28

Physical Basis for E

- Material is an assembly of atoms in a stable configuration





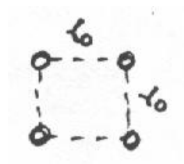
Slope of F vs. γ curve at γ_o

$$S_o = \left(\frac{dF}{d\gamma}\right)_{\gamma=\gamma_o}$$

For small values of γ near the equilibrium, force required to move an atom to distance γ is given as

$$F = S_o(\gamma - \gamma_o) \qquad \text{(Linear Behaviour)}$$

Force per unit area, i.e., stress, $\sigma = \dfrac{F}{\gamma_o^2}$



Such that $$\sigma = \frac{S_o}{\gamma_o^2}(\gamma - \gamma_o)$$

$$\sigma = \frac{S_o}{\gamma_o}\left[\frac{(\gamma - \gamma_o)}{\gamma_o}\right]$$, where $\left[\dfrac{(\gamma - \gamma_o)}{\gamma_o}\right]$ is equal to strain

$$\sigma = \frac{S_o}{\gamma_o}E$$ [Compare with $\sigma = E\varepsilon$]

This means $E = \dfrac{S_o}{\gamma_o}$

- E is a measure of the resistance of separation of adjacent atoms
- E primarily depends on the slops (S_o) of F vs. γ relation at equilibrium spacing γ_o (ideal behaviour)
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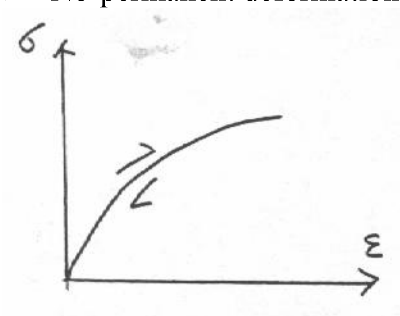
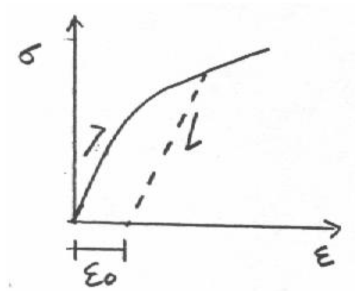
Non-Linear Elasticity

- Stress is a not-linear function of strain
- Loading and unloading pasts are the same
- No-permanent deformation after unloading



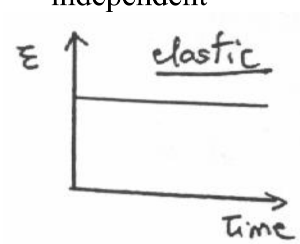
Elastoplastic Behaviour

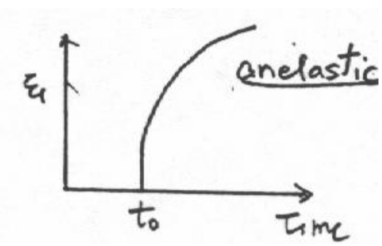
- Non-linear σ-ε relation
- Permanent deformation (ε_o) after unloading
- Eg. Metals at high stresses



Anelasticity

- Up to this point, elastic deformation of a stressed material is consider to be time-independent





- In some materials, strain continues to increase with time when subject to a constant stress. The time dependant elastic behaviour is called *ANELASTICITY*
- Eg. Polymers