Updating Pavement Deterioration Models Using the Bayesian Principles and Simulation Techniques

Feng Hong
Graduate Research Assistant, Department of Civil Engineering
The University of Texas, ECJ 6.510, Austin, TX 78712
Phone: (512) 232-6598, Email: fenghong@mail.utexas.edu

Jorge A. Prozzi
Assistant Professor, Department of Civil Engineering
The University of Texas, ECJ 6.112, Austin, TX 78712
Phone: (512) 232-3488, Fax: (512) 475-8744,
Email: Jorge.Prozzi@engr.utexas.edu

Total word count: 6450

Submission date: July 7, 2005

ABSTRACT

The research presented in this paper analyzes and updates an existing incremental pavement deterioration model, which was originally developed based on data from the AASHO Road Test. Material and structural properties, environmental effects and traffic loading, the three main major factors dominating the characteristic of pavement performance, are incorporated into the model.

When panel data are available from large field experiments, due to the limited number of variables that can be controlled and observed, unobserved heterogeneity is almost inevitable. Most of the existing models did not fully account for the heterogeneity issue. During this research, focus is placed on the heterogeneity of the individual model parameters. For this purpose, the Bayesian approach is adopted for its ability to address the issue of interest. Unlike most traditional model estimation approaches based on sampling techniques and normality assumption, the Bayesian approach aims to obtain realistic parameter distributions through a combination of existing knowledge (prior) and updated information from the data collected. The Gibbs sampling algorithm with Monte Carlo Markov Chain (MCMC) simulation is applied to estimate parameter distributions.

It was found that there is significant variability in the parameters. Hence the need exists to address heterogeneity in modeling pavement performance. Furthermore, it is shown that not all estimated parameters are normally distributed as commonly assumed. It is therefore suggested that the performance model developed in this research provides a more realistic forecast than most previous models. In addition, pavement deterioration forecast based on the Gibbs output is performed at different percentile levels with varying inspection frequencies, which can enhance the decision-making process in pavement management.

In general, the Bayesian approach presented in this paper provides an effective and flexible alternative for model updating, which can be applied to both the data from road test sites and other data sources of interest.

INTRODUCTION

Pavement performance prediction plays a central role in highway infrastructure system design and management. Performance can be expressed in terms of distresses including rutting, cracking, and roughness. It can also be evaluated through subjective indicators such as the present serviceability rating (PSR) as established by AASHO during the Road Test (HRB, 1962).

To-date, much effort has been given to developing state-of-the-art pavement performance models to address the deterioration process. Basically, either the empirical or mechanistic approach, or the two approaches combined, is utilized for performance modeling. For example, an empirical sigmoid curve was applied to fit the pavement deterioration process by Garcia-Diaz and Riggins (1984). A mechanistic approach was incorporated to develop the damage functions for rutting, fatigue cracking, and loss of pavement serviceability index (PSI) by Rauhut et al. (1983). After the World Bank road test in Brazil, Paterson (1987) established a series of empirical performance models on the basis of a comprehensive study of previous modeling efforts and the characteristic of the road test data. Currently, the most widely accepted model is the American Association of State Highway Transportation Officials (AASHTO) design equation (AASHTO, 1993).

In addition, in terms of model format, both linear and nonlinear models were examined in the previous studies. The nonlinear models were found to be more appropriate for determining deterioration due to traffic and environmental impacts. However, many of the nonlinear models lack physical explanation, statistical soundness, or are not suitable for the deterioration process. These problems have been improved through the development of the nonlinear model with panel data (Archilla, 2000; Archilla and Madanat, 2001; Prozzi, 2001; Prozzi and Madanat, 2003). In such cases, heterogeneity not previously observed across the pavement sections was captured by the intercept term by means of random-effect models, while it is assumed that the other parameters were fixed. Although this approach produces efficient parameter estimates, heterogeneity is not entirely captured. Pavement performance heterogeneity should potentially be reflected not only through the intercept but through the regression parameters. Hence, it is more reasonable to relax the above assumption and let the parameters be random (random parameters model), since each section may possess unique characteristics affecting the deterioration process. The tradeoff of adopting more flexible model is the need for additional computational effort. Even for the random-effect nonlinear model, the process of searching for an iterative solution under Generalized Least Square (GLS) is time-consuming. However, an optimum solution is not guaranteed. It can be expected that the computational work required for a random parameter nonlinear model is more demanding. As an alternative, this paper will describe how the Bayesian approach can be applied to effectively address the issue.

The Bayesian approach offers the flexibility to incorporate existing knowledge so that previous experience can be utilized rather than ignored (Zellner, 1971). In addition, obtaining the distribution of the parameters as random variables to reflect the performance heterogeneity (the main objective of this study) is straightforward, and the output is the density function, which can provide comprehensive statistics of the individual parameters.

The following section describes the proposed incremental pavement deterioration model. Section 3 presents the Bayesian approach to estimating the specified model. The Gibbs sampling technique is utilized to estimate the distribution of each parameter. In addition, the performance forecast procedure is examined based on Gibbs output and an illustrative example is given. The final section presents study conclusions and major findings.

MODEL SPECIFICATION

AASHO Model

Based on the experimental data from the AASHO Road Test, a state-of-the-art pavement performance model was established (HRB 1962), as is shown in Equation 1:

$$p_t = p_0 - (p_0 - p_f)\left(\frac{W}{\rho}\right)^{\alpha} \quad (1)$$

Where

p_t: serviceability at time t

p_0: serviceability at time $t = 0$, i.e. initial serviceability

p_f: terminal serviceability

W: accumulated axle load repetitions until time t

ρ: accumulated axle load repetitions until failure

α: regression parameter determining the curvature of performance model

The deficiency of the model was identified with regard to determining ρ and α in both specification and parameter estimation aspects (Rauhut et al., 1983; Prozzi and Madanat, 2000). Consequently, an improved model is adopted by considering the factors affecting pavement performance, in particular those factors involving pavement structures, environment, and traffic.

Since the performance deterioration trend is described by the AASHO model, it can be utilized as the basis for an improved model; thus the proposed model is formed from the AASHO equation:

$$p_t = f(N_t) + \mu = a - bN_t^{\,c} + \mu \quad (2)$$

Where

p_t: serviceability at time t

N_t: some measure of traffic until time t

μ: error term

a: parameter representing initial serviceability

b: parameter representing the deterioration rate

c: parameter representing the curvature

Improved Model Specification

The deterioration of pavement is dependent on the combined impact of traffic load and environment on the pavement system. Therefore, as a sound model, Equation 2 should incorporate these relevant variables. Since traffic load is directly included in the term N_t, the remainder of the variables (environment and pavement-related input) are incorporated in the term b.

As proposed in the original AASHO model, the thickness index $1+\gamma_1 H_1+\gamma_2 H_2+\gamma_3 H_3$ denotes the contribution of each pavement layer to the resistance of the deterioration, where γ_1, γ_2, and γ_3 are the parameters of each pavement layer that denote the relative capability of the individual layer to support the traffic. The three layers are surface, base, and subbase with thicknesses of H_1, H_2, and H_3, respectively. The term "1" did not have a physical meaning, but it is used to avoid the possible mathematic indetermination when the thickness index is equal to zero since it appears in the denominator of the function for α (HRB, 1962). In the current model, a different form is proposed to make the specification physically meaningful. This is based on the assumption that pavement performance deterioration rate will decrease with the increase in the pavement structural strength and vice versa. Consequently, the contribution to the resistance of deterioration from the pavement structural strength is depicted as:

$$b_S = \exp\{\lambda\}\exp\{\gamma_1 H_1+\gamma_2 H_2+\gamma_3 H_3\} \tag{3}$$

The implication is that γ_1, γ_2, and γ_3 will be expected with negative signs. Except for the three thickness items reflecting the pavement surface layer, base, and subbase, it is shown that Equation 3 includes one multiplicative term $\exp\{\lambda\}$. The reason for adding this term is to consider the contribution of the subgrade. Physically, $\exp\{\lambda\}$denotes the pavement deterioration rate due to first unit of traffic applied directly on top of the subgrade.

Environmental impact on pavement performance is another critical aspect since pavement is sensitive to temperature and moisture, which is not directly reflected in the AASHO model. It was found that the most significant environmental factor influencing pavement performance is the frost penetration gradient (Prozzi, 2001). The frost penetration gradient, denoted as G_t, is defined as the ratio between the change of frost penetration depth during period *t* and the length of time. Hence, the environmental factor, denoted as b_E, is:

$$b_E = \exp\{\varphi G_t\} \tag{4}$$

Where

φ: parameter to be estimated

Intuitively, during the thawing period, with the drop in the frost penetration depth (i.e., negative gradient), the serviceability loss will accelerate due to the sudden loss of base and subbase strength. Thus, the sign of φ is expected to be negative so that during this period φG_t will be larger than zero leading to b_E being larger than one, which implies more serviceability loss.

For the complete form, an incremental model is adopted. The benefits of using an incremental model to forecast the next-period serviceability loss are:

1. from the infrastructure management context, the one-period-forward (next-period) performance forecast is usually adopted in decision-making, allowing for the adjustment of the schedule plan based on actual performance data; and
2. only next-period traffic is included while traffic from the previous traffic levels have already been obtained.

Based on previous research (Archilla, 2000; Prozzi, 2001), the first order Taylor expansion on Equation 2 based on the one-period-forward performance condition is:

$$\Delta p_t = p_{t-1} - p_t = dN_{t-1}{}^{e}\Delta N + \varepsilon \quad (5)$$

Where

d, e: parameters to be estimated
N_{t-1}: some measure of accumulative traffic until the one-period backward of time t
ΔN: projected incremental traffic during the time period between t-1 and t
ε: error term

The determination of d follows. From the first order Taylor expansion, it is shown that d (Equation 5) is equal to b multiplied by c (Equation 2). b encompasses both the structural factor b_S and environmental factor b_E. In addition, considering the sign of c should be nonnegative, it is integrated with $\exp\{\lambda\}$ as $\exp\{\lambda'\}$. Hence, the denotation of d becomes:

$$d = \exp\{\lambda'\}\exp\{\gamma_1 H_1 + \gamma_2 H_2 + \gamma_3 H_3\}\exp\{\varphi G_t\} \quad (6)$$

Consequently, the incremental model representing serviceability loss at certain pavement sections can be expressed in the full specification as:

$$\Delta p_{it} = \exp\{\beta_0 + \beta_1 H_{1i} + \beta_2 H_{2i} + \beta_3 H_{3i} + \beta_4 G_t\} N_{i,t-1}^{\beta_5} \Delta N_{it} + \varepsilon_{it} \quad (7)$$

$$N_{i,t-1} = \sum_{l=1}^{t-1} \Delta N_{il} \quad (8)$$

$$\Delta N_{il} = n_{il}\left\{\left(\frac{FA_i}{\beta_6 18}\right)^{\beta_8} + A_i\left(\frac{SA_i}{18}\right)^{\beta_8} + B_i\left(\frac{TA_i}{\beta_7 18}\right)^{\beta_8}\right\} \quad (9)$$

Where

Δp_{it}: serviceability loss in pavement section i during time period t
$\beta_0 \sim \beta_8$: parameters to be estimated
H_{1i}: surface layer thickness at pavement section i
H_{2i}: base layer thickness at pavement section i
H_{3i}: subbase layer thickness at pavement section i
G_t: frost penetration gradient at time period t
$N_{i,t-1}$: cumulative traffic until the one-period backward of time t
ΔN_{il}: incremental traffic during the time period between l-1 and l
ε_{it}: error term
n_{il}: traffic volume during the time period between l-1 and l
FA_i: front axle load magnitude
SA_i: single axle load magnitude
TA_i: tandem axle load magnitude
A_i: number of single axles on one vehicle

B_i : number of tandem axles on one vehicle

Three types of axle configurations were applied during the AASHO Road Test: front steering axles (single axle with single wheels), single axle with dual wheels, and tandem axles with dual wheels. It is assumed that the impact on pavement performance with each pass of axle load can be converted into an equivalent value based on its configuration and magnitude. The standard load for single axle with dual wheels is 18 kips (80 kN), while two coefficients β_6 and β_7 are assigned for obtaining the standard loads of the front axle and tandem axles, respectively.

MODEL ESTIMATION THROUGH BAYESIAN APPROACH

Equation 7 is nonlinear in the parameters. To capture the unobserved heterogeneity of pavement performance, the parameters are regarded as random variables across the different pavement sections. It will be shown that the Bayesian approach with MCMC simulation is remarkably powerful for estimating the statistics of these variables. Bayesian inference combines the information from observed data with prior knowledge about the parameters (prior) to arrive at the updated distribution of the parameters (posterior) (Zellner, 1971), which is described as:

$$p(\underline{\theta}|\underline{X},\underline{Y}) = \frac{p(\underline{X},\underline{Y}|\underline{\theta})p(\underline{\theta})}{\int p(\underline{X},\underline{Y}|\underline{\theta})p(\underline{\theta})d\underline{\theta}} \propto p(\underline{X},\underline{Y}|\underline{\theta})p(\underline{\theta}) \tag{10}$$

Where

$p(\underline{\theta}|\underline{X},\underline{Y})$ denotes the posterior distribution of the set of parameters $\underline{\theta}$ given the observed data including $\underline{X}$ (explanatory variables) and $\underline{Y}$ (dependent variable)

$\underline{\theta}$ consists of the regression parameters and the parameters for the error term in Equation 7

$p(\underline{X},\underline{Y}|\underline{\theta})$ denotes the likelihood of the observed data given the parameters $\underline{\theta}$

$p(\theta)$ denotes the prior distribution of the parameter set $\underline{\theta}$

Due to the integral in the denominator (i.e. total probability across the parameter space being a constant), the posterior can be written as proportional to the numerator, the multiplication of likelihood and prior distribution, as shown in the right-hand side of Equation 10.

The Prior Specification

Customarily, it can be assumed that the prior distributions of the regression parameters are independently and normally distributed (Gelfand et al., 1990). The prior distribution for the regression parameters in the proposed model specification (see Equations 7, 8, 9), $\underline{\beta} = [\beta_0, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7, \beta_8]^T$ can be denoted as:

$$\underline{\beta} \sim N(\underline{\beta_u}, \Lambda)$$

Where

$\underline{\beta_u}$ is the mean of the parameters, denoted as $[\beta_{u0}, \beta_{u1}, \beta_{u2}, \beta_{u3}, \beta_{u4}, \beta_{u5}, \beta_{u6}, \beta_{u7}, \beta_{u8}]^T$.

Λ is the variance-covariance matrix of parameter vector $\underline{\beta}$.

When determining the statistics for the prior, previous research results are used as the learned knowledge on the issue of interest. However, if there is no existing information available, an estimate based on the authors' judgment is proposed.

First, the mean values of prior parameters are established. A prior mean of β_0 = -1 is assumed, corresponding to the value of "1" in the denominator of α in the AASHO equation. For β_1, β_2, and β_3, according to the AASHO equation, the three parameters -0.44, -0.14, and -0.11, respectively are used as the prior means. For β_4, using the information in Prozzi (2001), an estimated value is proposed as -0.1 for prior mean of β_4. For β_5, a similar estimate was made based on Prozzi's findings so that the prior mean is assumed to be -0.5. Since β_6 is used to convert the steering axle load into the equivalent load of a single axle with dual wheels, 18 kip (80 kN), its prior mean is assumed to be 0.5. Following similar reasoning, the prior mean for β_7 is assumed to be 2.0 since it is the case for the tandem axle. For β_8, considering the "4th power law" was used in estimating the load pavement impact (Huang, 2003), the prior mean is assumed to be 4.0. In summary, the prior means for the parameters are chosen as:

$$\underline{\beta_u} = [-1, -0.44, -0.14, -0.11, -0.1, -0.5, 0.5, 2, 4]^T$$

Second, concerning the uncertainty of each parameter, the concept of precision is applied. In Bayesian approach, the precision — denoted as τ — is customarily defined as the reciprocal of variance, namely, $\tau = \frac{1}{\sigma^2}$. It should be noted that varying precision levels may result in different posterior distributions. Hence, three representative precisions are selected in this study to examine the sensitivity of the posterior distribution to different assumptions on prior precisions. The first alternative adopts relatively smaller precision, 0.1, corresponding to the higher uncertainty of the parameters. The second alternative uses relatively larger precision, 1, corresponding to the lower uncertainty. When choosing the third precision alternative, it is assumed that before the study the parameter of variance (CoV) for each parameter is 1 (i.e., $\sigma / \mu = 1$). In addition, the error term ε_{it} is also assumed to be independently and normally distributed,

$$\varepsilon_{it} \sim N(0, \eta)$$

Where

η is the precision of the error term distribution. Considering that the majority of the model's uncertainty is incorporated in the error term and little is known about it before the study, the error term's precision is further reflected by a gamma distribution:

$$\eta \sim gamma(\phi, \varsigma)$$

Where

ϕ, ς: the parameters of the gamma distribution of η

As a result, by assuming that the priors are independent, the prior joint distribution including all the parameters is obtained as:

$$p(\theta) \propto \prod_{k=0}^{8} \sqrt{\frac{\tau_{k,k}}{2\pi}} \exp\left(\left(-\frac{\tau_{k,k}}{2}\right)(\beta_k - \beta_{uk})^2\right)\eta^{\phi-1} \exp\{-\varsigma\eta\} \tag{11}$$

Where

$\tau_{k,k}$: the k,k element of matrix Λ^{-1}, which is the precision of the distribution of β_k

η: the precision of the error term distribution

The Likelihood Function

As stated previously, the error term ε_{it} is assumed to be independently and normally distributed. Hence, the likelihood function is:

$$p(\underline{X},\underline{Y}|\underline{\theta}) = \prod_{i=1}^{n}\prod_{t=1}^{T_i} \sqrt{\frac{\eta}{2\pi}} \exp\left\{\left(-\frac{\eta}{2}\right)(\Delta p_{it} - g_{it}(\underline{X},\underline{\beta}))^2\right\} \tag{12}$$

Where

n is the number of pavement sections

T_i is the number of time periods when data were collected on pavement section number i

Δp_{it} is the observed serviceability loss during time period t on pavement section i

η, as in Equation 11

$g_{it}(\underline{X},\underline{\beta}) = E(\Delta p_{it}|\underline{X}) = \exp\{\beta_0 + \beta_1 H_{1i} + \beta_2 H_{2i} + \beta_3 H_{3i} + \beta_4 G_t\} N_{i,t-1}^{\beta_5} \Delta N_{it}$

The Posterior

With the prior and likelihood functions at hand, using Equation 10, the posterior is obtained as:

$$\begin{aligned} p(\underline{\theta}|\underline{X},\underline{Y}) \propto & \prod_{i=1}^{n}\prod_{t=1}^{T_i} \sqrt{\frac{\eta}{2\pi}} \exp\left\{\left(-\frac{\eta}{2}\right)(\Delta p_{it} - g_{it}(\underline{X},\underline{\beta}))^2\right\} \\ & \times \prod_{k=0}^{8} \sqrt{\frac{\tau_{k,k}}{2\pi}} \exp\left\{\left(-\frac{\tau_{k,k}}{2}\right)(\beta_k - \beta_{uk})^2\right\}\eta^{\phi-1} \exp\{-\varsigma\eta\} \end{aligned} \tag{13}$$

As shown in the posterior distribution, there are ten parameters to be estimated. Although the joint distribution of the ten variables conditional on the given data is obtained, the goal is to arrive at the marginal distribution of each variable, which requires the multi-dimensional integration of the right-hand side of Equation 13. An alternative to avoid the complexity in obtaining the marginal distribution is available through the Gibbs sampler with MCMC simulation, which is presented in the following section.

Implementation of the MCMC

Gibbs sampling is a Markovian updating scheme (Gelfand and Smith, 1990). The process of the algorithm used in the Gibbs sampling is described as follows: for a set of random variables U_1, U_2, …, U_m, the joint distribution is denoted as $f(U_1, U_2, ..., U_m)$. With given arbitrary starting values of $U_s{}'s$, say $U_1^{(0)}$, $U_2^{(0)}$, …, $U_m^{(0)}$, the first iteration of random draws of $U_s{}'s$ is obtained as:

$U_1^{(1)}$ from $f\left(U_1 \middle| U_2^{(0)}, U_3^{(0)}, \ldots, U_m^{(0)}\right)$

$U_2^{(1)}$ from $f\left(U_2 \middle| U_1^{(1)}, U_3^{(0)}, \ldots, U_m^{(0)}\right)$

$\vdots$

$U_m^{(1)}$ from $f\left(U_m \middle| U_1^{(1)}, U_2^{(1)}, \ldots, U_{m-1}^{(1)}\right)$

In a similar manner, the second set of random draws of $U_s{}'s$ is obtained through the update process. After r iterations as shown above, the series of $U_s{}'s$ is obtained as $\left(U_1^{(r)}, U_2^{(r)}, \ldots, U_k^{(r)}\right)$. It is shown that under mild conditions for each variable $U_s^{(r)} \xrightarrow{d} U_s \sim f(U_s)$ as $r \to \infty$ (Geman and Geman, 1984), which means that after enough iterations, *r*, , $U_s^{(r)}$ can be regarded as a random draw from the distribution of $f(U_s)$.

Based on the above algorithm, the application of MCMC for obtaining the marginal distribution of each parameter conditional on observed data is straightforward. It is shown that the joint distribution by the given data set is available through the Bayesian approach, which is the posterior (Equation 13). With the joint conditional distribution of the parameter set, the MCMC simulation is carried out, leading to the simulated distribution of each parameter of interest.

PARAMETER ESTIMATION RESULTS

Through applying the modeling process and estimation methodology aforementioned to the AASHO Road test data, the basic sample statistics (mean and standard deviation) of the parameters are presented in Table 1. The statistics of each parameter are almost the same among the three precision levels. The results imply that the posterior is not significantly sensitive to the prior's selected precision in this case.

As the representative, the parameter sets from the alternative level with prior precision equal to 1 are chosen for further analysis (it is similar for the remaining two). In addition to the means and standard deviations shown in Table 1, the following statistics are investigated in Table 2: 1) coefficient of variance (CoV), 2) skewness, and 3) kurtosis. Meanwhile, the densities of the various parameters are illustrated in Figure 1.

With regard to the parameters of three pavement thicknesses, it is shown that the largest mean relative performance deterioration resistance is denoted by $-\beta_1$ (0.485), followed by $-\beta_2$ (0.157), and then $-\beta_3$ (0.153). This result follows the practice in pavement engineering that the surface layer contributes more to the resistance of serviceability loss than the base and subbase. Moreover, it is implied that the relative contribution of the surface layer's unit thickness is around three times that of the base or subbase. The base and subbase are close in terms of their ability to resist serviceability loss. In addition, Table 2 shows that CoV of the base and subbase are close to each other, with the surface layer being slightly higher. Skewness and kurtosis values of the three parameters being near 0 and 3, respectively, suggests that each of the distributions can be regarded as normal, which is also reflected in their densities as shown in Figure 1.

Other parameters deserving special attention are those associated with traffic information, β_6, β_7, and β_8. β_6 and β_7 are the coefficients for estimating equivalent axle loads for the steering and tandem axles, respectively. The mean of β_6 equal to 0.686 means that the equivalent load for a steering axle is 12.3 kip (54.8 kN) on average, while β_7 equal to 2.229 implies that a tandem axle load is 40.1 kip (178 kN). The larger CoV value of β_6 (24 percent) suggests that the equivalent damage estimation for steering axle load has a larger variability than that for the tandem axle load (CoV of 9 percent). In addition, both distributions are significantly asymmetric, as shown by their densities in Figure 1. Both positive skewness values indicate that the equivalent loads are more concentrated on lighter axle loads for the steering and tandem axle. The mode of β_6 equal to 0.511 implies that the most popular steering axle equivalent load is 9.2 kip (40.8 kN), which differs significantly from the mean value. A similar result applies to the coefficient for determining the equivalent load for a tandem axle. Table 2 shows that the kurtosis values of β_6 and β_7 are larger than 3, particularly that for β_7, indicating that the distributions of the two parameters' are more outlier-prone than the normal distribution. Therefore, the assumption of normality of the two parameters is not recommended. Regarding β_8, its mean value of 3.22 is different from its counterpart obtained in the analysis from the original AASHO Road Test, which is around 4. The result implies that by applying the "4th power law" the impact of heavy loads on pavement might be overestimated, while the impact of a light load is underestimated. Additionally, both skewness (close to 0) and kurtosis (close to 3) confirm the normal assumption of β_8 as in the traditional analysis.

For the remaining parameters, the statistics in Table 2 and the densities in Figure 1 suggest they be symmetric and normal distributions.

PAVEMENT PERFORMANCE FORECAST

The ultimate goal for model estimation is to forecast pavement performance. As shown in the following paragraphs, performance forecast is a straightforward process. A simulation approach based on the random parameter variables obtained in Gibbs output is adopted to achieve the predicted pavement performance.

Let $\Delta p_{n+1,t}{}^{f}$ denote the serviceability loss of a pavement section number $n+1$ (outside the observed n samples) during the time period t to be forecast, which is given by:

$$\Delta p_{n+1,t}{}^{f} = \exp\{\beta_0 + \beta_1 H_{1,n+1} + \beta_2 H_{2,n+1} + \beta_3 H_{3,n+1} + \beta_4 G_t\} N_{n+1,t-1}^{\beta_5} \Delta N_{n+1,t} + \varepsilon_{n+1,t} \qquad (14)$$

Where, the explanatory variables are known for the pavement section of interest and the parameters are from the post-convergence of the Gibbs output. The error term is assumed random draw obtained from simulated errors of the Gibbs output.

In order to obtain the forecast serviceability loss for each time period, Monte Carlo simulation is again carried out as follows:

1. Obtain one set of random draws of $\underline{\beta}$, denoted as $\underline{\beta}^{(k)}$, from the post-convergence Gibbs output, plug them into Equation 14 and calculate $\Delta p_{n+1,t}{}^{f(k)}$, which has the normal density:

   $$\Delta p_{n+1,t}{}^{f(k)} \sim N\left(g_{n+1,t}\left(\underline{X}_{n+1}, \underline{\beta}^{(k)}\right), \tau^{(k)}\right) \tag{15}$$

   Where

   $\underline{X}_{n+1}$ is the vector of explanatory variables from the pavement section $n+1$ to be forecast, $\tau^{(k)}$ is the precision.

2. Obtain one random draw from the density of $\Delta p_{n+1,t}{}^{f(k)}$

3. Repeat steps (1) and (2) to obtain a total of K values.

4. Calculate the statistics of the forecast serviceability loss based on the sample of K random values obtained in steps (1), (2) and (3). For example, the predicted mean of serviceability loss can be estimated through the sample mean:

   $$E\left(\Delta p_{n+1,t}{}^{f} \middle| \underline{X}_{n+1}, Data\right) = \frac{1}{K}\sum_{k=1}^{K} \Delta p_{n+1,t}{}^{f(k)} \tag{16}$$

A Case Study of Performance Forecast

To illustrate the effectiveness of the model, pavement section 271 from the AASHO Road Test is selected, a replicate section that was not used during the development and estimation of the model.

The serviceability loss at each time period is calculated to arrive at the performance deterioration curve. Figure 2 represents the forecast deterioration curve by simulated mean values at time points along the road test duration. It is demonstrated that the forecast line fits the observations accurately.

From a pavement management perspective, one-period-forward performance deterioration forecast is of prime interest. In general, pavement condition data are collected on a regular basis at frequencies determined by the highway agency based on resource availability. Thus, pavement performance forecasts between two data collection intervals are required for decision-making. To illustrate the effect of survey frequency and prediction confidence, six possible scenarios are presented as a case study: 1) Pavement condition survey frequency: every two years, once a year, and every six months, and 2) Prediction confidence: 50 and 60 percentile.

The performance forecast in the first interval is based on the initial condition and that for the second interval is based on the observed PSI at the beginning of that interval. The percentile-based incremental performance forecast results for the three groups of scenarios with increasing frequency are depicted in Figures 3, 4, and 5, respectively. The observed performance curve lies close to the 50-percentile lines. In addition, figures indicate that as the confidence level increases (higher percentile), forecast performance drops significantly, which leads to a profound implication for reliability-based pavement design. The reliability obtained from the median performance forecast line corresponds to the condition 50 percent of system reliability. The 10-percent increase in reliability level can be obtained but results in a significant drop in forecast

PSI. Moreover, it is implied that with an increase in inspection frequency, the forecast variation decreases, leading to more confidence in the performance forecast. Therefore, it is implied that the balance between accurate performance forecast and inspection frequency is of critical importance in order to realize optimal highway infrastructure system management. Figures 3 through 5 indicate that, when all sources of uncertainty are properly accounted for, aiming at higher reliability levels (such as 90 percent or 95 percent) may be unrealistic. In-depth research into determining appropriate and realistic reliability levels as a function of inspection frequency (function of available resources at the highway agency) is imperative.

CONCLUSIONS

An incremental pavement deterioration model was investigated in this paper based on the AASHO Road Test data. The model accounts for the fundamental factors associated with pavement performance: structural properties, environment, and traffic.

The Bayesian approach is used to update the model parameters. The purpose is to assess the effects of unobserved heterogeneity on pavement performance model parameters by exploring the characteristic of the variability of parameters. The parameter information from previous studies and the authors' understanding of the deterioration process are used as the prior. The posterior is obtained by updating the prior with the observed data and reflects the characteristics of the actual deterioration process. The Gibbs sampling technique is used to estimate the distributions of the individual parameters. Through the application of this methodology and based on the critical analysis of the model and the prediction results, the following results should be considered:

1. The model was developed in an attempt to capture the physical process of deterioration and the estimated results match data and the validation set accurately. The relative magnitudes are also consistent with previous research.
2. The normality assumptions seem reasonable for most parameters with the exception of β_6 and β_7, which are used for calculating the equivalent axle load for steering and tandem axle, respectively.
3. The parameter $\beta_8 = 3.22$ is slightly lower than the counterpart in the "4th power law" obtained after the original analysis of the AASHO Road Test data, which suggests that the impact of heavier loads (>18 kips) on the pavement is currently overestimated, while the impact of the lighter axle loads is underestimated.
4. Their standard deviations and CoV show that there is significant variability in the individual parameters. This means that the model is subject to uncertainty through the variability of the parameters, leading to the heterogeneity of the pavement performance across sections. Consequently, it implies that addressing unobserved heterogeneity in pavement performance is a critical issue in modeling.

In addition, the Gibbs output is used for predicting the pavement performance. As an advantage of the incremental model, pavement performance forecast at a given interval (corresponding to the inspection frequency) can be obtained. Furthermore, the varying percentiles of performance forecast are presented, which can aid the decision-making process based on the confidence in the forecast variables.

Following the philosophy of Bayesian updating, the research approach presented in this paper can be applied to enhance the current model as new data is collected. With the collected data from different sources (such as Pavement Management Systems), the results obtained herein can serve as the prior. After integrating with the new data through the approach presented in this paper, the updated model can be used to facilitate pavement management at particular locations of interest.

REFERENCES

AASHTO. (1993). AASHTO Guide for Design of Pavement Structures, Washington, D.C.

Archilla, A. R. (2000). *Developing of Rutting Progression Models by Combining Data from Multiple Sources*. PhD dissertation, University of California, Berkeley, CA.

Archilla, A. R., and Madanat, S. M. (2001). “Development of Pavement Rutting Model by Combining Data from Different Experimental Sources.” *Journal of Transportation Engineering*,, 126(4), pp. 291–299.

Garcia Diaz, A. and Riggins, M. (1984). “Serviceability and distress methodology for predicting pavement performance.” *Transportation Research Record 997*, Transportation Research Board, National Research Council, Washington, D.C., pp. 56–61.

Gelfand, A. E. and Smith, A. F. M. (1990). “Sampling-Based Approaches to Calculating Marginal Densities.” *Journal of the American Statistical Association*, 85, pp. 398–409.

Gelfand, A. E., Hills, S. E., Pacine-poon, A. and Smith, A. F. M. (1990). “Illustration of Bayesian Inference in Normal Data Models using Gibbs Sampling.” *Journal of the American Statistical Association*, 85, pp. 972–985.

Geman, S. and Geman D. (1984). “Stochastic Relaxation, Gibbs Distributions and the Bayesian Restoration of Images.” *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 6, pp. 721–741.

Highway Research Board (HRB). (1962). *The AASHO Road Test*. Special Report Nos. 61A through 61G and 73, Washington, D.C.

Huang, Y. H. (2003). *Pavement Analysis and Design*. 2nd Ed. Prentice Hall, Inc., New Jersey.

Paterson, W. D. O. (1987). *Road Deterioration and Maintenance Effects: Models for Planning and Management*, The Highway Design and Maintenance Series, Johns Hopkins University Press, Baltimore, MD.

Prozzi, J. A. and Madanat, S. M. (2000). “Analysis of Experimental Pavement Failure Data using Stochastic Duration Models.” *Transportation Research Record 1699*, Transportation Research Board, National Research Council, Washington, D.C.

Prozzi, J. A. (2001). *Modeling Pavement Performance by Combining Field and Experimental Data*. PhD dissertation, University of California, Berkeley, CA.

Prozzi, J. A., and Madanat, S. M. (2003). “Incremental Nonlinear Model for Predicting Pavement Serviceability.” *Journal of Infrastructure Systems*, 129(6), pp. 635–641.

Rauhut, J. B., Lytton, R. L., Jordhal, P. R. and Kenis, W. J. (1983). “Damage functions for rutting, fatigue cracking and loss of serviceability in flexible pavements.” *Transportation Research Record 943*, Transportation Research Board, National Research Council, Washington, D.C., pp. 1–9.

Zellner, A. (1971). *An Introduction to Bayesian Inference in Econometrics.* John Wiley & Sons Inc., New York.

LIST OF TABLES AND FIGURES:

Table 1. Estimation Results at Three Prior Precision Levels

Precision	$\tau = 0.1$		$\tau = 1$		CoV=1	
Parameters	Mean	StdDev.	Mean	StdDev.	Mean	StdDev.
β_0	-5.781	0.029	-5.432	0.028	-5.457	0.027
β_1	-0.454	0.044	-0.485	0.044	-0.479	0.048
β_2	-0.157	0.015	-0.157	0.016	-0.157	0.016
β_3	-0.151	0.014	-0.153	0.014	-0.151	0.014
β_4	-0.117	0.006	-0.121	0.006	-0.120	0.006
β_5	-0.244	0.031	-0.265	0.030	-0.265	0.028
β_6	0.723	0.172	0.686	0.168	0.768	0.259
β_7	2.164	0.125	2.229	0.190	2.220	0.228
β_8	3.030	0.238	3.222	0.238	3.167	0.241

Table 2. Statistics on Variability of Parameters and Normality Check

Parameters	CoV	Skewness	Kurtosis
β_0	-0.01	0.026	2.78
β_1	-0.09	0.038	2.91
β_2	-0.10	0.030	3.11
β_3	-0.09	-0.012	3.00
β_4	-0.05	-0.128	2.99
β_5	-0.11	0.113	2.79
β_6	0.24	1.15	4.65
β_7	0.09	2.04	12.0
β_8	0.07	0.089	2.99

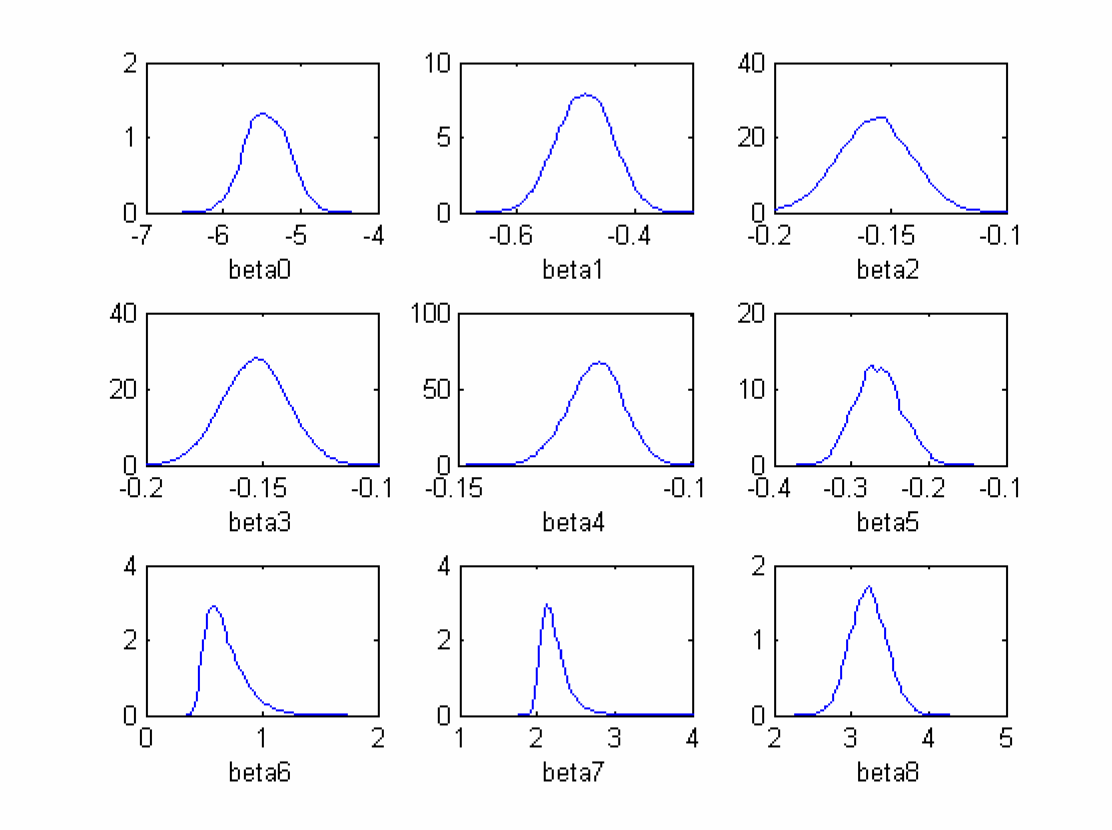

Figure 1. The Densities of Estimated Parameters

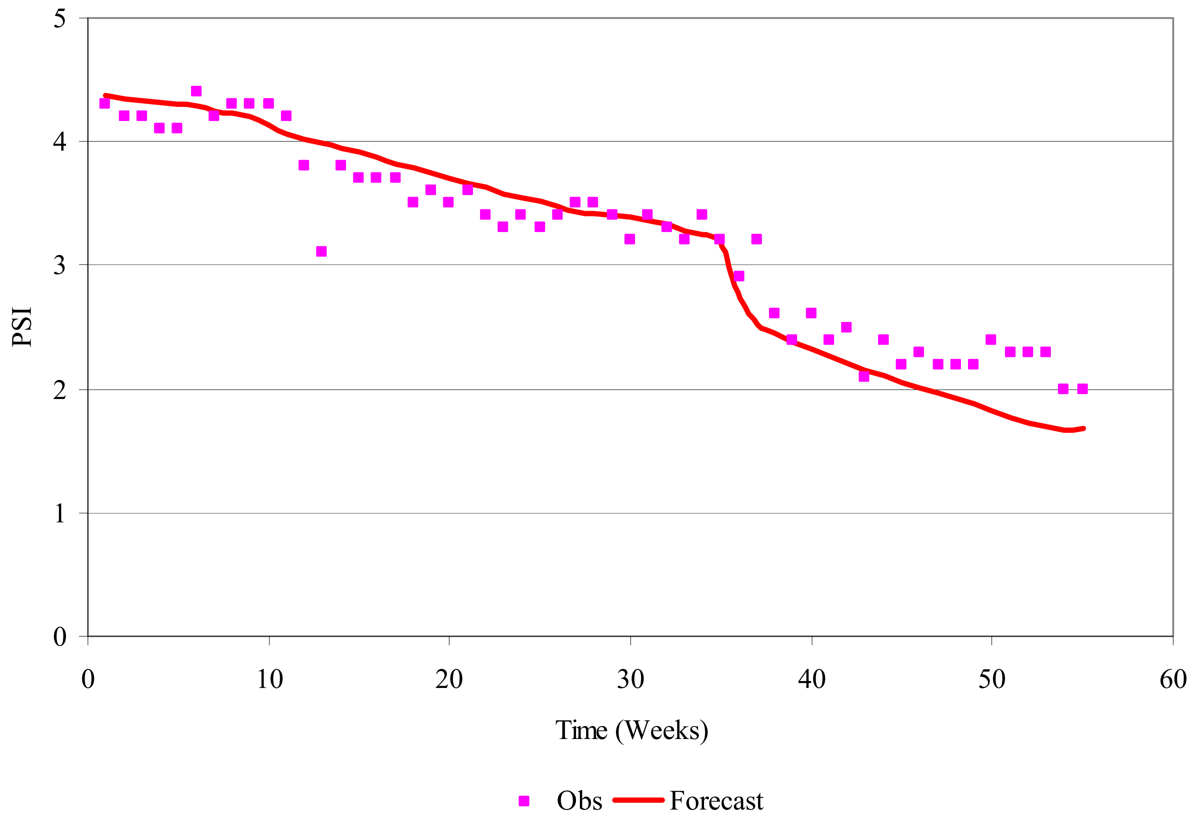

Figure 2. Observed versus Forecast Mean Performance Curves on Section not Used in Model Estimation Sample

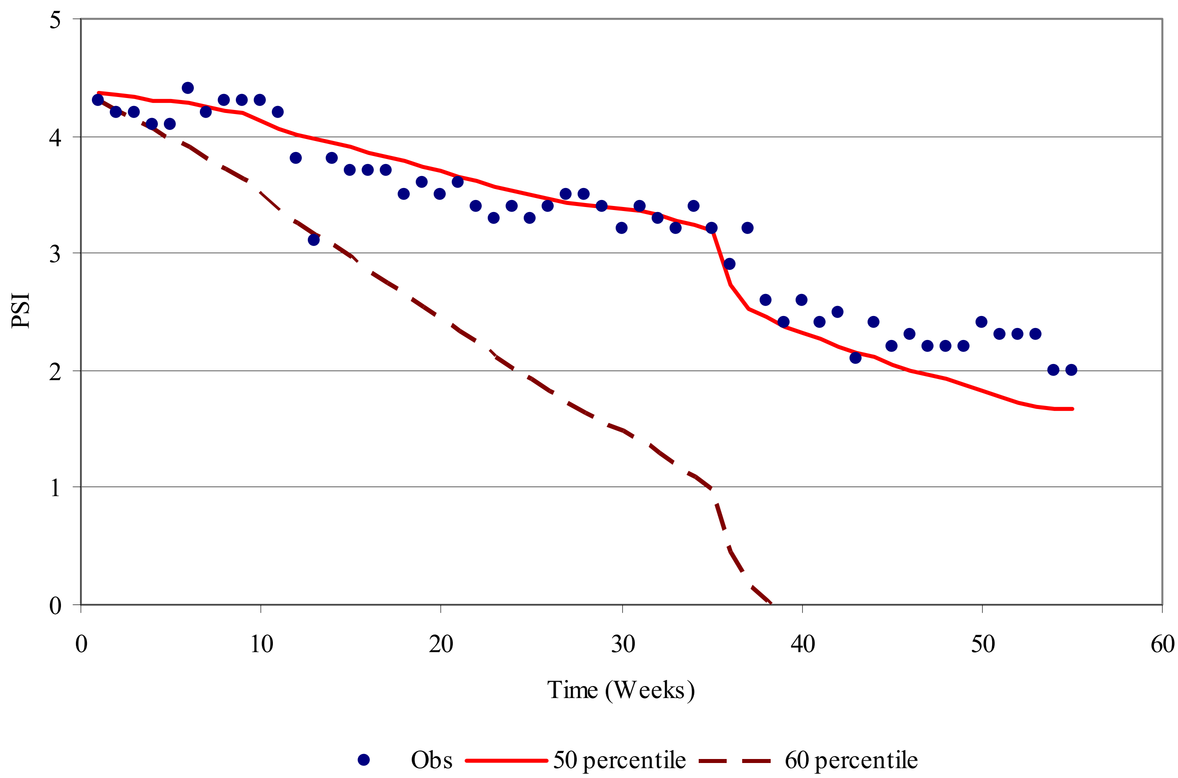

Figure 3. Observed versus Forecast Performance at Different Percentile Levels with an Inspection Frequency of Once Every Two Years

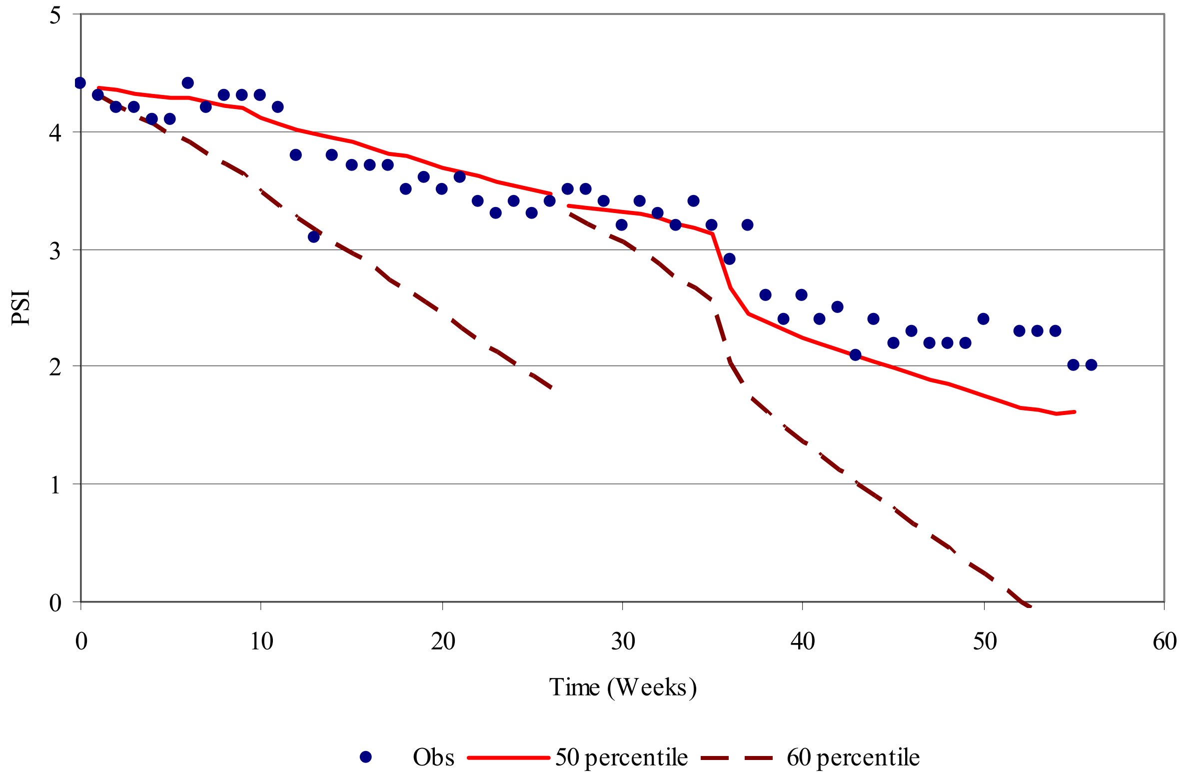

Figure 4. Observed versus Forecast Performance at Different Percentile Levels with an Inspection Frequency of Once per Year

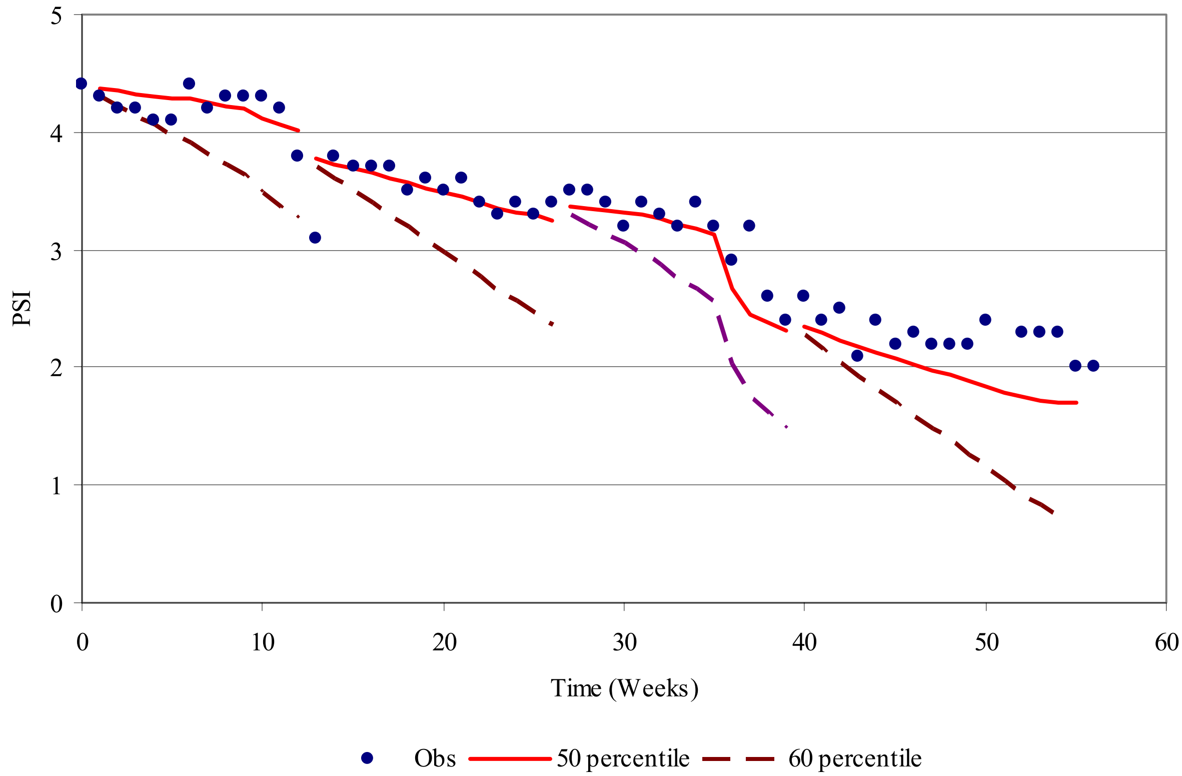

Figure 5. Observed versus Forecast Performance at Different Percentile Levels with an Inspection Frequency of Once per 6 Months